1958

A nuclear magnetic resonance device for classifying grounded mines

Armstrong, John E.

Monterey, California. Naval Postgraduate School

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

A NUCLEAR MAGNETIC RESONANCE

DEVICE FOR CLASSIFYING GROUNDED MINES

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John E. Armstrong

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A NUCLEAR MAGNETIC RESONANCE DEVICE FOR CLASSIFYING GROUNDED MINES

by

John E. Armstrong

Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
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G. R. Giet.

Chairman
Dept. of Electronics

Approved: R S Glasgow

Academic Dean

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ABSTRACT UNCLASSIFIED

This thesis is a study of the feasibility of classifying submerged mines by means of a nuclear magnetic resonance device which observes the gradients in the perturbed magnetic field of the earth in the vicinity of the mine.

The author's work on this project was accomplished at Varian Associates Instrument-Research Laboratory in Palo Alto, California, during the period January to March, 1958, while a student in the Engineering Electronics curriculum at the U. S. Naval Postgraduate School, Monterey, California.

The idea for this means of classifying mines originated with Dr. Martin E. Packard of Varian Associates.

The author wishes to express his appreciation to Dr. Packard, and Messrs. Dolan Mansir and John Drake of Varian Associates, and to Professor Carl E. Menneken of the U. S. Naval Postgraduate School for their help, suggestions and encouragement.

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TABLE OF SYMBOLS

$\bar{a}$ = nuclear spin angular momentum

I = nuclear spin quantum number

h = Planck's constant = 6.625×10^{-34} joule-sec

e = electronic charge = 1.602×10^{-19} coulomb

M = mass of nucleus

c = speed of light = 2.998×10^{8} meters/sec

M_n = Nuclear magneton = 5.05×10^{-27} joule-meter^2/weber

g = nuclear g factor

γ_p = proton gyromagnetic ratio = 2.7×10^{4} (gauss-sec)$^{-1}$

$\bar{H}$ = magnetic field (induction) in gauss (1 gauss = 10^{-4} weber/meter^2)

γ = magnetic field (induction) in gamma (1 gamma = 10^{-5} gauss)

$\bar{H}_p$ = polarizing magnetic field

$\bar{H}_e$ = earth's magnetic field (average value = 50,000 γ)

H = change of magnetic field across a sample

μ = nuclear magnetic moment of one nucleus

$\bar{M}$ = macroscopic nuclear magnetic dipole moment per unit volume (the total resultant of all the nuclear moments of a substance) ($\bar{M} = \chi\bar{H}$)

χ = magnetic susceptibility = 3.4×10^{-10} gauss/oersted for protons in water

ν = Larmor frequency

ω = angular precession frequency

Φ = magnetic induction flux, webers (1 weber = 10^{8} maxwell)

T_1 = relaxation time (time constant with which $\bar{M}$ exponentially approaches its final value when sample is being polarized)

T_2 = transverse relaxation time (time constant at which $\overline{M}$ decays from its polarized value to zero if the sample is in a zero gradient field.)

T = time constant at which $\overline{M}$ decays from its final polarized value to zero if the sample is in a non-zero gradient field

1. Introduction.

Presently operational mine-detecting sonar cannot distinguish with any degree of certainty between an actual mine and a non-mine which gives echoes similar to a mine. Since it is obviously desirable to be able to distinguish a mine from a rock, mound, sunken wreckage, steel barrel, etc., a means of classifying such sonar contacts is one of the present needs of the Fleet.

If sonar of high enough resolving power were available to determine the physical shape of the contact, the contact could be classified by this means. Any other device that could determine the shape of the unknown object could also be used to classify it as a mine or a non-mine. In place of the technique of high resolution sonar to determine the shape of the object, the possibility of detecting its shape by detecting the pattern of the magnetic field gradient around it has been suggested.(1) The earth's magnetic field is constant at any given instant over a portion of space comparable to the volume occupied by a mine. If, however, the field is perturbed by the presence of the mine, it will no longer be a constant field but will possess gradients. A look at the gradients of the magnetic field in the vicinity of a sonar contact might give enough information to determine the approximate shape.

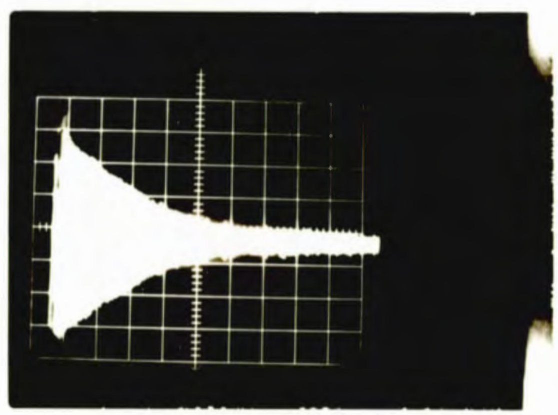
The thought of measuring the gradients around a mine to determine its shape, and a method of making these measurements originated with Dr. Martin E. Packard of Varian Associates, Palo Alto, California. He found a method whereby the large gradients around a mine could be observed by means of a free nuclear precession magnetometer. The free precession magnetometer was developed by Varian, Packard and Bowen(2)

for measuring the magnitude of the earth's magnetic field. It generates as its output an audio frequency which is proportional to the magnitude of the magnetic field in which the sensing element is located. As the result of an "interrogation" of the instrument, an audio frequency output signal is generated which decays exponentially in a few seconds. (Figure 1a) The frequency of this audio signal must be measured before the signal dies out in order to determine the magnitude of the field. (This is not a continuous reading instrument, but gives single readings once every few seconds.)

A gradiometer had been developed previously by Varian which utilized the principle of the free precession magnetometer. This gradiometer measured the gradient of a magnetic field by means of two sensing elements located a fixed distance apart. The magnitude of the magnetic field was measured simultaneously at both points and the difference in the readings divided by the distance between the two sensing elements gave the value of the gradient of the field. This gradiometer was for the purpose of measuring much smaller gradients than those encountered near a mine so it could not be used to measure the larger gradients present near a mine. The distance between its sensing elements was so large that the fine shape of field of non-constant gradient between the sensing elements could not be detected. In addition, in a field of high gradient, the value of field could change significantly from one side of a sensing element to the other and thereby "kill" the signal.[1] To measure the high gradients encountered around a mine, a single element gradiometer was needed which would measure the change in field across the sensing

[1]This effect is discussed in Section 2f and on p 31 of Section 3.

a $T = 408$ Milliseconds
Gradient $= 5.8\ \gamma$/inch

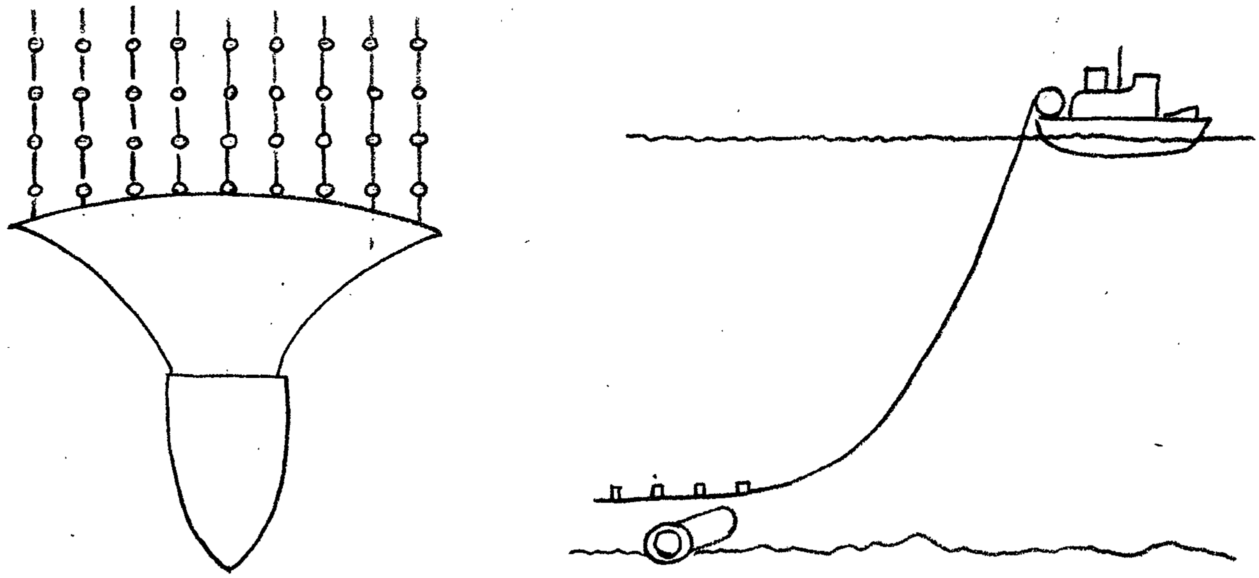


b. $T = 280$ Milliseconds
Gradient $= 14.2\ \gamma$/inch

Figure 1 Two Examples of Free Nuclear Precession Signal from a Sample in a Gradient Field.

element. Such an instrument was found possible when it was observed that the characteristic decay time of the signal from an earth's field magnetometer was shortened considerably if a magnetic material was held near the single sensing element. (See Figure 1b) The nearer the metal object was held to the sensing element, the shorter the decay time became. This was attributed to the increased gradient of magnetic field near the metal. It is shown in Appendix I that the decay time is related to the change in the magnitude of the magnetic field across the sensing element. This means that gradients large enough to give a significant change in field across a small sensing coil could be measured by using a single sensing head and measuring the decay time of the output signal. The value of gradients present around a mine are of about the right magnitude to make the magnetometer a good prospect as a measuring device of these gradients. The great need for a mine classifying device and the facility with which the magnetometer could be modified for this use made it seem worthwhile to investigate the feasibility of this method of mine classification.

If an operational system were built to classify sonar contacts the information that would be required from the system would be simultaneous measurement of the values of the field gradient at several points around the contact. This could be achieved with a modified version of a free nuclear precession magnetometer. Instead of one sensing element, it would be provided with several, arranged in a plane array in the water over the contact. (See Figure 2) Each sensing element would provide information to a display device which would produce a visual display of the gradients at each point in the array. This display might be, for



(a) Plan View (b) Side View

Figure 2. A Possible Arrangement for a Towed Array of Sensing Elements.

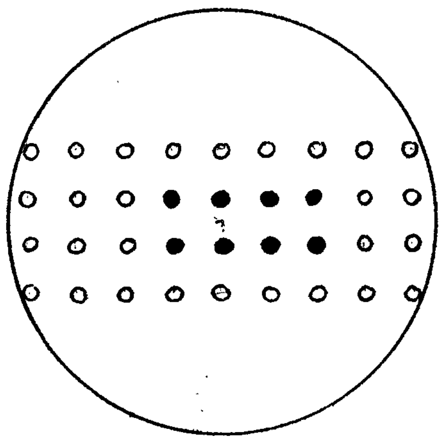


Figure 3. A Cathode Ray Oscilloscope used for display of information from the towed array of sensing elements. An array of spots on the face of the scope would correspond to the physical array of sensing elements. The intensity of each spot would be proportional to the magnetic gradient at the corresponding sensing element.

instance, an array of spots on a cathode ray oscilloscope corresponding to the array of sensing heads, each spot of intensity proportional to the magnetic field gradient at that point in the physical array. (See Figure 3) This would have the effect of presenting a mosaic picture of the contact. The physical array of sensing elements could be in the form of a matrix whose elements are, say, six to twelve inches apart. The matrix could contain, perhaps, 100 sensing elements and be towed behind a minesweeper in the fashion of regular sweep gear. If this matrix were towed across a sonar contact which appears on sonar as a possible mine, the visual display of the magnetic gradients that the sensing matrix encountered might provide a quick evaluation of the contact or at least provide information in addition to that obtained from the sonar, and permit a reduction in the number of contacts that would have to be investigated further. It should be reiterated here for emphasis that this proposed device is for the purpose of classifying mines, not detecting them.

A proposal by Varian Associates to the Bureau of Ships[1] suggested a study in two parts of the feasibility of this method of classification. The first part was a study as to whether the signature of a mine contained enough information to identify it, and the second part was the study and development of a system prototype for evaluation if the mine signature were found to be a feasible means of identifying mines. This thesis will be concerned with the first part of that study.

The system which was proposed to the Bureau of Ships is built around a free nuclear precession magnetometer. A better understanding of what follows will be obtained if a brief description of a free precession magnetometer is presented. A more detailed explanation of the physical

principles involved in the operation of a magnetometer will be found in the next section on Theory. A brief, qualitative description of the magnetometer, follows here:

Since the formulation of the physical concept of nuclear spin by Pauli, several devices have been built which utilize this principle. The free nuclear precession magnetometer is one such instrument. The operation of a free nuclear precession magnetometer depends on the gyroscope-like properly of free protons. This gyroscope-like property results from the fact that each proton spins about an axis through its center. In the case of water, or a light hydrocarbon, the presence of ionized hydrogen provides free hydrogen nuclei, or, free protons. In a manner analogous to a force acting on the spin moment of an ordinary gyroscope and causing it to precess, the force of the earth's magnetic field acts on the magnetic moment of the spinning protons and causes them to precess. The frequency of precession in both the gyroscope and the proton is directly proportional to the magnitude of the force acting on it. Therefore, if the frequency of proton precession could be measured, the magnitude of the force causing the precession, that is, the magnitude of the earth's magnetic field, would be determined.

In order to measure the frequency of proton precession, many protons must be caused to precess coherently so that an effect will be obtained which is large enough to observe. In the free nuclear precession magnetometer, this coherence of proton precession is obtained as follows: The sample, such as water, containing free protons, is placed inside a coil which is fed with a direct current great enough to produce a static magnetic field of approximately one hundred gauss inside the coil. This

one hundred gauss field, about two hundred times the magnitude of the earth's magnetic field, is strong enough to cause a large number of spinning protons to line up with their axes all pointing in the direction of the one hundred gauss polarizing field. The coil is oriented such that the polarizing field is approximately perpendicular to the earth's field. The direct current in the coil is then cut off abruptly and the polarizing field collapses. This leaves the earth's magnetic field present, and so the protons acted on by this force, begin to precess about the direction of the earth's magnetic field. Since the protons began this precession from the same starting position, they start out precessing in phase. A large number of protons precessing in phase in the sample is enough to induce an audio voltage in the coil surrounding the sample. This voltage varies at the frequency of precession. The frequency is measured and converted into terms of field strength of the earth's magnetic field. The signal obtained has a shape like that shown in Figure 1. The audio frequency voltage is contained in an exponentially decaying envelope. The magnetometer is instrumented to measure this frequency before the signal dies out and thereby give a measure of the earth's magnetic once each time the sample protons are polarized and permitted to precess.

2. Theory.

The method of operation of a free precession magnetometer and a method of adapting it to measure gradients may be described more fully by means of certain physical concepts. This section presents a resume of the pertinent physical concepts which are used to provide a quantitative description of the operation of the magnetometer.

a. Nuclear Spin

The concept of nuclear spin was first suggested by Pauli. It came as the culmination of a long search for a suitable theory to explain the presence of a hyperfine structure observed in some of the spectral lines of many elements. The presence of these hyperfine lines was assumed to be due to additional dicrete energy levels in the atom, but the phenomena providing these additional energy levels could not be discovered. Then Pauli theorized that the nuclei of the atoms spin about an axis and therefore possess an angular momentum

$$\bar{a} = \bar{I} \frac{h}{2\pi} \qquad (1)$$

where $\bar{I}$ is the "nuclear spin quantum number."
The hyperfine lines could then be explained by the energies possessed by the spinning nuclei.

b. Nuclear Magnetic Moments (3).

The nucleus has an angular momentum due to its spin. The spinning charge of the nucleus also gives it a magnetic moment. Methods have been developed to determine the value of nuclear magnetic moments. The nuclear magnetic moment is expressed in terms of a "Nuclear Magneton," Mn

defined by

$$Mn = \frac{eh}{4\pi Mc} \qquad (2)$$

Then, analogous to Landé's g factor for electrons, a nuclear g factor may be defined:

$$g = \frac{\text{Nuclear Magnetic Moment in units of Mn}}{\text{Nuclear Spin Angular Momentum in units of } \hbar} \qquad (3)$$

$$g = \frac{\mu}{I\,Mn}$$

A similar definition is that of the gyromagnetic ratio, γp

$$\gamma p = \frac{\text{Nuclear Magnetic Moment}}{\text{Nuclear Spin Angular Momentum}} \qquad (4)$$

$$\gamma p = \frac{\mu}{I\hbar}$$

Therefore, the Nuclear Magnetic Moment, μ is given by:

$$\mu = g\,I\,Mn = g\,I\,\frac{eh}{4\pi Mc} = \gamma p\ I\hbar \qquad (5)$$

By Larmor's Theorem, a nucleus of magnetic moment $\bar{\mu}$ subjected to a constant homogeneous field $\bar{H}$ will precess about the direction of $\bar{H}$ with a frequency

$$\nu = \frac{\mu H}{I h} \qquad (6)$$

$$\omega = \frac{\mu H}{I\hbar} = \gamma p\ H$$

So, for a known constant H, the value of μ can be determined by measuring the precessional frequency, ν. One method termed the "Magnetic Resonance Method" developed by Rabi[1] and his co-workers depends upon resonance between the precession frequency of the nuclear "magnet" about a constant

[1] I. I. Rabi, S. Millman, P. Kusch, and J. R. Zacharias Phy Rev 53, 318 (1938); 55 526 (1939)

magnetic field direction and an impressed high frequency magnetic field.

c. Nuclear Induction

Very accurate measurements of nuclear magnetic moments were made in 1946 by Bloch, Hansen and Packard(3) using a process called "Nuclear Induction." This was a significant modification of the magnetic resonance principle. A diagram of the apparatus they used is shown below:

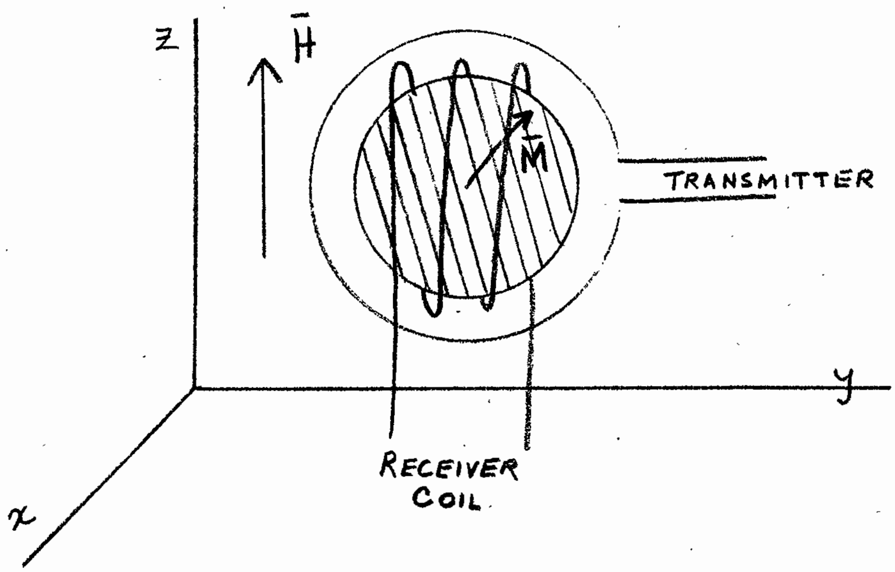


Figure 4. A Diagram of the Apparatus used in the Nuclear Induction Experiment by Bloch, Hansen and Packard.

A spherical sample of the material under investigation is placed between the faces of a large magnet which provides a strong, homogenous, constant magnetic field in the Z-direction. Many of the nuclear magnets will line up with this field; that is, they will precess about the Z-direction at the Larmor frequency, but each proton will have a component of its magnetic moment, $\bar{\mu}$ in the direction of $\bar{H}$. Instead of dealing with microscopic quantities, it is possible to consider the sample as a paramagnetic substance having a large moment, $\bar{M}$ located at the center of the spherical sample. $\bar{M}$ also has a component in the Y-direction, so a

Y-component of magnetic flux exists. An emf will be induced in the receiver coil anytime the magnetic flux in the Y-direction changes. A flat coil in the Y-Z plane is fed by a high frequency generator thereby setting up an alternating magnetic field in the X-direction. If the frequency of this field, H_x, is far removed from the Larmor frequency, its action on the precessing protons will be negligible because it quickly loses phase coherence with the protons. But as the frequency approaches the Larmor frequency, H_x will interact more strongly with the precessing protons and cause $\bar{M}$ to make a larger angle with the Z-direction as it precesses about $\bar{H}$. This causes a much larger change in magnetic flux in the Y-direction, the flux change being greater as the high frequency field approaches the Larmor frequency and interaction becomes stronger. Therefore the voltage induced in the receiving coil exhibits a resonance at the Larmor frequency. This action can be described analytically as follows:

If the fields existing are

$$H_x = 2 H_1 \cos \omega t$$
$$H_y = 0 \qquad (7)$$
$$H_z = H_0$$

then let H be the resultant of H_0 and H_1

If $\bar{a}$ = angular momentum

$\bar{T}$ = Total torgue

$$\text{then } \frac{d\bar{a}}{dt} = \bar{T} \qquad (8)$$

$$\text{and } \bar{T} = (\bar{M} \times \bar{H}) \qquad (9)$$

where $\bar{M}$ is the macroscopic nuclear magnetic moment per unit volume.

$$\bar{M} = \gamma_P \bar{a} \qquad (10)$$
$$\bar{a} = \frac{\bar{M}}{\gamma_P}$$

$$\frac{d\bar{a}}{dt} = \frac{1}{\gamma_\rho} \frac{d\bar{M}}{dt} = (M \times H) \qquad (11)$$

$$\frac{d\bar{M}}{dt} = \gamma_\rho (M \times H) \qquad (12)$$

This time variation of $\bar{M}$ results in a voltage being induced in the receiving coil. It is seen that the nearer the angle between $\bar{M}$ and $\bar{H}$ approaches 90° the larger will be the voltage induced. Maximum voltage (or resonance) occurs when $\bar{M}$ is perpendicular to $\bar{H}$ (or approximately perpendicular to $\bar{H}_o$).

The nuclear magnetic moment is determined by measuring frequency of resonance and applying the relation

$$\nu = \frac{\mu H}{I h} \qquad (6)$$

It is seen that an accurate determination of μ depends on an accurate knowledge of the value of H, the constant magnetic field. Since the value of μ for many different nuclei has been accurately determined, Bloch suggested that this method could be reversed and used to determine values of unknown static magnetic field.

d. Free Nuclear Precession

Techniques of magnetic resonance and nuclear induction have been used for precision measurements of fields of 25 gauss and higher. An entirely new concept, free nuclear precession, made it possible to extend field strength measurements down to the order of the earth's magnetic field, 0.5 gauss (50,000 gamma) and measure them to an accuracy of 0.1 gamma.

Free nuclear precession was first demonstrated in 1954 by Russell Varian, Martin Packard and Lt. Chas. Bowen, USN. It utilizes the methods of nuclear induction but uses a fundamentally different way of producing

coherent precession of the magnetic moment about the magnetic field which is being measured. In the magnetic resonance method, coherent precession is obtained by impressing an r.f. magnetic field perpendicular to the measured field. This causes the macroscopic moment, $\bar{M}$, to tend to orient perpendicular to the measured field (i.e., parallel to the r.f. field). When the frequency of this r.f. field is near the Larmor frequency, $\nu = \gamma_p H$ the magnetic moment, $\bar{M}$, is tipped from its original direction and precesses about the measured field, thereby inducing a measurable voltage in the receiving coil.

However, in the free nuclear precession scheme, $\bar{M}$ is lined up perpendicular to the field being measured, i.e., the earth's magnetic field. This is accomplished by placing it in a strong d.c. polarizing field oriented perpendicular to the earth's field. The polarizing field is then suddenly turned off, leaving $\bar{M}$ perpendicular to $\bar{H}_e$, the earth's magnetic field. The force of $\bar{H}_e$ acting on $\bar{M}$ causes $\bar{M}$ to precess about the direction of $\bar{H}_e$ at the Larmor frequency given by $\nu = \gamma_p \bar{H}_e$. The precessing moment will induce a voltage in a coil oriented with its axis perpendicular to the direction of $\bar{H}_e$ and the frequency of this voltage can be measured to determine the magnitude of $\bar{H}_e$. Figure 5 shows the orientation of $\bar{M}$ during polarization and during free precession. The coil need not be exactly perpendicular to the earth's field, but that orientation will give the maximum signal. Any varying component of $\bar{M}$ which is parallel to the coil axis will induce a voltage. The induced voltage is maximum when the coil is perpendicular to the earth's field.

The voltage induced in the pick-up coil by the precessing moment is given by the relation:

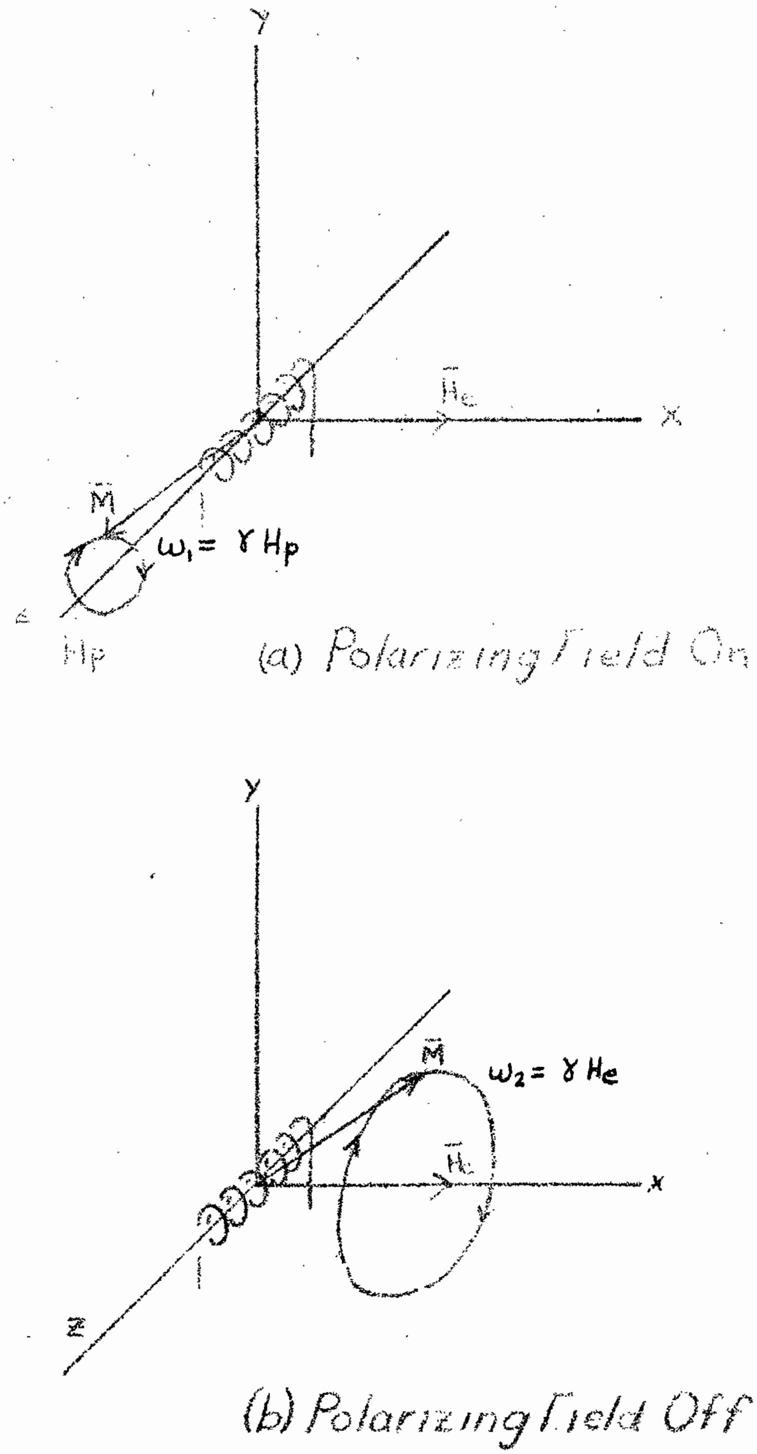


Figure 5 Orientation of the Macroscopic Nuclear Magnetic Moment, $\bar{M}$, in the Free Nuclear Precession Process.

$$V = -NA\chi H_p \omega \sin \omega t \times 10^{-8} \text{ volts} \tag{13}$$

where N = number of receiver coil turns

A = cross sectional area of sample

χ = susceptibility of water sample

$= 3.4 \times 10^{-10}$

H_p = polarizing field in gauss

$\omega = \gamma_p H_e = 2.7 \times 10^4 H_e$

This expression for the induced voltage is derived as follows:

$$V = N \frac{d\phi}{dt}$$

$$\phi = A M \cos \omega t \quad \text{where } M = \chi H_p$$

$$\phi = A \chi H_p \cos \omega t$$

$$\frac{d\phi}{dt} = -\omega A \chi H_p \sin \omega t$$

$$N \frac{d\phi}{dt} = -N A \chi H_p \omega \sin \omega t \qquad \text{Abvolts}$$

$$V = -N A \chi H_p \omega \sin \omega t \times 10^{-8} \quad \text{volts}$$

e. Transverse Relaxation Time

The accuracy of the field measurement is dependent upon how long the induced signal of the free precession is present for measurement. This signal does not remain indefinitely, but decays in a given time, T_2 . (T_1, the relaxation time is the time constant with which $\bar{M}$ exponentially approaches its final value after the polarizing field is turned on. T_1 is of the same order of magnitude as T_2 and is always equal to or greater than T_2). The signal decays approximately exponentially with the time constant T_2 because of phase incoherence caused by inhomogenities of the field across the sample. These are caused by local fields from spin-spin coupling and spin-lattice coupling.

f. Effect of External Field Gradient on Relaxation Time

The signal induced in a coil by free precession of nuclei decays

approximately exponentially with time constant T_2. This assumes that the external field across the sensing head is a homogeneous field such as the earth's magnetic field. However, if a gradient exists in the magnetic field across the sample, the signal will decay in a shorter time, T. An expression for this shortened decay time, T is:

$$\frac{1}{T} = \frac{1}{T_2} + \frac{1}{T_{2*}} \qquad (14)$$

See Appendix I

where $T_{2*} = \frac{2}{\gamma_p \Delta H}$ (15)

γ_p is the gyromagnetic rato, 2.7×10^4 $(\text{gauss-sec})^{-1}$

ΔH is the change in field, in gauss, across the sample.

Combining equations (14) and (15) and solving for ΔH

$$\Delta H = \frac{2}{\gamma_p} \left(\frac{T_2}{T} - 1 \right) \frac{1}{T_2}$$

Since T_2 is a constant of the material used as the sample in the sensing head, a measurement of the decay time, T, is sufficient to determine the change in field across the sample. This combined with a knowledge of the sample dimensions in the direction of the gradient gives the value of the gradient. (The direction of the gradient is inconsequential if a spherical sample is used since the dimensions of the sample are then the same in any direction.)

The envelope of the signal from a sample in a gradient field is only approximately exponential as is shown in Appendix I. However, within experimental accuracy, the envelope may be considered a pure exponential of time constant T when measuring T.

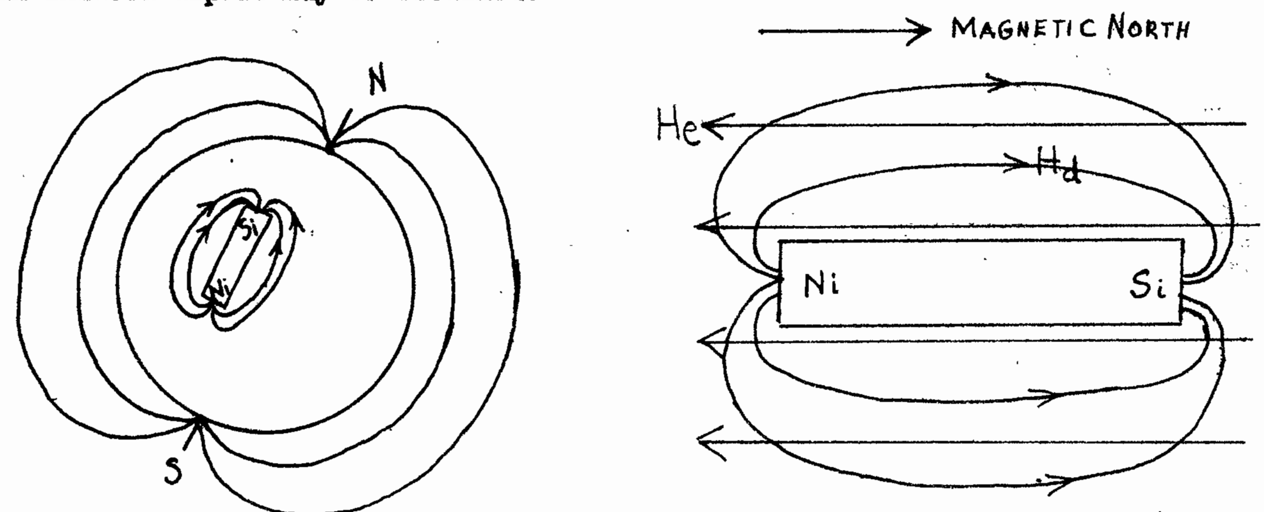
It is seen that if the <u>gradient</u> of a magnetic field is to be determined, it is not necessary to measure the <u>frequency</u> of the free precession

signal; only the time of decay of the free precession signal has to be measured.

g. Dipole Perturbation of the Earth's Magnetic Field

A mine case which is approximately cylindrical in shape will possess induced magnetism from the earth's field and will perturb the earth's field in a manner similar to a magnetic dipole. For this reason a knowledge of the shape of the field around a dipole will give an idea of what shape field should be expected around a mine case. In addition, the shape of the field around a dipole must be known since, to classify a mine, the characteristics in its field shape which differ from the field of a dipole must be detected. That is, if the mine is to be identified by its characteristic field shape, details of the actual mine shape must cause a field pattern different from the characteristic dipole pattern (which could be expected from an object such as a steel barrel.)

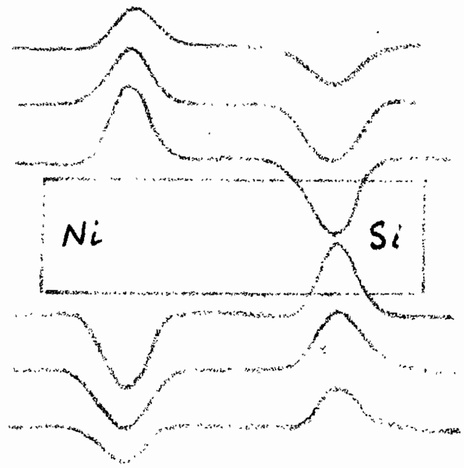
With the aid of Figure 6, the approximate shape of the field around an induced dipole may be obtained:



(a) Magnetism induced in a ferrous material in the earth's magnetic field.

(b) Magnified view of induced dipole in (a)

Figure 6. A Heuristic Derivation of the Magnetic Field in the Vicinity of an Induced Magnetic Dipole.



(c) Resultant total field H in the vicinity of induced magnetic dipole.

Figure 6. A Heuristic Derivation of the Magnetic Field in the Vicinity of an Induced Magnetic Dipole.

If the shape of the field around a dipole is that shown in Figure 6 it can be visualized that, at a given height above the dipole, the gradient in the field is higher at the ends of the dipole than in the center. Therefore, the gradient pattern in a plane above a mine would be expected to contain points of high gradient over the ends (when oriented in a North-South direction) and lower gradients over the center. If this configuration of the gradient field is plotted, it would appear as in Figure 7.

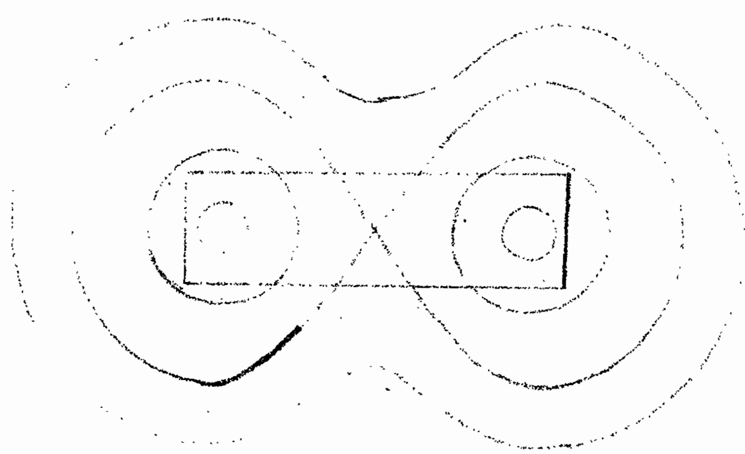


Figure 7. A Plot of the Expected Magnetic Gradients in a Plane Above an Induced Magnetic Dipole.

A development by Rempel[1] shows that the shape of the field obtained heuristically in Figure 6 is a good approximation of the correct shape. His derivation of expressions for the field above an induced dipole are contained in Appendix II.

[1] R. C. Rempel, Dipole Perturbation of the Earth's Magnetic Field, Varian Associates Technical Memorandum, TMO-23.

3. Experimental Work.

As was stated in the introduction, the purpose of this thesis is to obtain data and interpret it in order to determine the feasibility of classifying mines by means of a nuclear magnetic resonance (NMR) device. The data required for this purpose is the value of the gradient of the earth's magnetic field at points in a plane just above a mine case. With a plot of this data, a map of constant gradient lines can be drawn. Also a mosaic plot to simulate the gradient field information as it would be displayed on a CRO may be made. These two plots will then serve as the basis of determining the feasibility of a system utilizing NMR classification. If the two plots resemble the mine case in shape or at least if they differ significantly from the plot obtained if a simple magnetic dipole were substituted for the mine case, then the proposed system will be considered feasible. If the plot is not different enough from a plot of the dipole to add any information to that available by sonar, the proposed system will be considered impracticable.

At a large distance from the mine case, little effect on the earth's field can be detected. A greater perturbation of the field will be observed closer to the mine, and to obtain a plot that shows any detail of the shape of the mine, it will probably be necessary to get very close to the mine, a distance comparable to the diameter of the mine. In an attempt to obtain this detail, gradients were measured in a plane twenty-four inches above the top surface of the mine as it lay flat on the ground. To obtain these measurements, the free precession signal was observed on an oscilloscope and a photograph taken of the signal. The value of the decay time was measured directly from the photograph by measuring the

time required for the signal to decay to $\frac{1}{\epsilon}$ times its initial value. A major problem associated with the experiment was the design and construction of a sensing head that would produce a signal as close to a large metal object as 24". Preliminary to the design of this sensing head, measurements of the magnitude of the field around the mine were made with a Varian Portable Magnetometer, M-49, for the purpose of determining what value of gradients to expect. With the sensing head, which contained a water sample 4" long and 1 3/4" in diameter, all signals disappeared within 40" from the mine and no reliable reading could be obtained within 60" of the mine. The signal disappears close to the mine because the change in field across the sample is so large that the decay time is too short to detect a signal. Therefore the sensing head must be made smaller so that the change of magnetic field across it is not so large. A smaller coil for the sensing head, however, introduces the following problems which must be overcome:

1. For the same polarizing current, a small coil produces a smaller polarizing field and therefore a smaller signal is available. To obtain the same value of polarizing field, a larger amount of power must be dissipated by the coil.
2. A smaller coil and sample will cause a reduction in $\frac{S}{N}$ ratio over that of a large coil and sample.[1]
3. A smaller sample in the coil will be able to produce

[1] F. Bloch, Nuclear Induction Phys. Rev. 70 (October 1946)

less signal than a larger sample because of the difference in the number of spins contributing to the signal.

To design and build a coil capable of giving a useful signal as close as 24" to a large metal object, was, then, the first step toward obtaining the data necessary for this study.

a. Coil Design

The selection of parameters of the coil used for a sensing head involves a compromise between those which give a relaxation time, T, that can be read fairly accurately, and those which provide a practicable $\frac{S}{N}$ ratio. In general the problem is this: a small coil is desired for a T long enough to read since in any magnetic gradient field $T \sim \frac{1}{\Delta H}$ where ΔH is the change of H across the sample. A large sample means a large ΔH and therefore a small T. On the other hand a large coil is desired for a large $\frac{S}{N}$. The voltage induced in the coil by precession of nuclear moments varies as the size of the coil, and this induced voltage must be large enough to override the various noise sources in the circuit.

Since large gradients are to be measured, the coil must of necessity be small. Calculations must be made, then, to determine just how small a coil may be used before $\frac{S}{N}$ becomes too small to be practicable.

b. Effect of Coil Size on $\frac{S}{N}$ Ratio

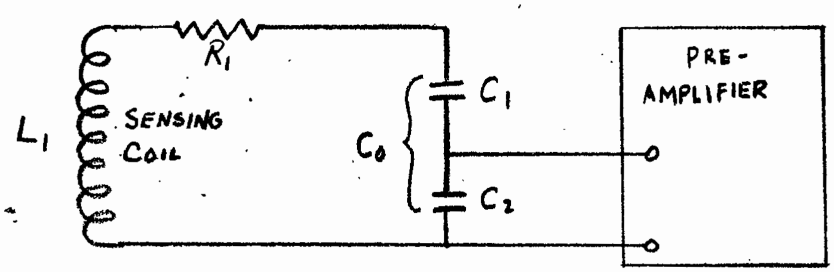


Figure 8. Input Circuit and Preamplifier

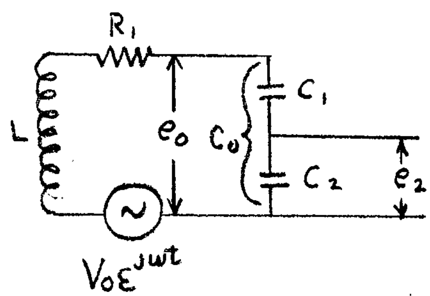
In the circuit used for this experiment (Figure 8) the precessing nuclear magnetic moment induces in the sensing coil, a voltage which is initially of the form $V_o \epsilon^{jwt}$. The $\frac{S}{N}$ ratio of the circuit can be found as follows:

$$\frac{S}{N} \text{ at amplifier output} = \frac{S}{N} \text{ at amplifier input} \times \frac{1}{\overline{NF}}$$

since the noise figure of the amplifier is defined as

$$\overline{NF} = \frac{\frac{S}{N} \text{ available at input}}{\frac{S}{N} \text{ available at output}} .$$

The $\frac{S}{N}$ available at the input is calculated as follows:



$$e_o = Q_1 V_o \varepsilon^{j\omega t}$$

$$e_2 = \frac{C_1}{C_1 + C_2} e_o$$

$$= \frac{C_1}{C_1 + C_2} Q_1 V_o \varepsilon^{j\omega t}$$

$$= V_2 \varepsilon^{j\omega t}$$

$\overline{V_p}^2$ = mean square noise voltage in series with

$\overline{V_c}^2$ = mean square noise voltage across C_1 C_2 combination

$\overline{V_a}^2$ = mean square noise voltage across input to amplifier

$= 4\ k\ TBRs \frac{C_1}{C_1 \not+ C_2}$ $= 4\ k\ TB\ (800)$ since the input circuit is designed so the amplifier sees 800 Ω looking into C_2.

V_2 = peak signal voltage across input to amplifier.

Then $\frac{S}{N}$ available at input $= \frac{P_s}{P_{nj}} = \left(\frac{\text{signal power}}{\text{noise power (due to Johnson noise)}} \right)$

$P_s = \frac{|V_2|^2}{2} G(f_o)$ $\quad G(f_o)$ = amplifier power gain at mid-frequency

$P_{nj} = 4kTB_{nj} R_2 G(f_o)$ $\quad R_2$ = resistance looking into C_2 (= 800 Ω)

and $\quad B_{nj}$ = Bandwidth of Johnson noise in c.p.s. which is taken to be

$\frac{\pi}{2}$ x the 3 db bandwidth of the input circuit.

$$|V_2|^2 = \left(\frac{C_1 Q_1 V_o}{C_1 + C_2} \right)^2 \quad \text{and} \quad R_2 = 800\ \Omega \text{ by design}$$

$$\therefore \frac{P_s}{P_{nj}} = \frac{\left(\frac{Q_1 C_1}{C_1 + C_2}\right)^2 V_o^2}{2 \times 4\, kTB_{nj}\,(800)} = \frac{V_o^2}{8kTB_{nj}} \quad \frac{(Q_1 C_1)^2}{800 (C_1 + C_2)^2}$$

$$\left.\frac{S}{N}\right]_{\text{OUT OF THE AMPLIFIER}} = \frac{V_o^2}{8kTB_{nj}} \left[\frac{1}{800 \left(\frac{C_1 + C_2}{Q_1 C_1}\right)^2} \times \frac{1}{\overline{NF}} \right] \qquad (1)$$

V_o = voltage induced in pick-up coil by precessing nuclear moments

Q_1 = Q of the pick-up coil

$\overline{NF}$ = noise figure of amplifier

C_1 and C_2 in series is the proper value of capacitance to resonate the pick-up coil.

The expression for V_o was derived by Rempel:[1]

$$V_o = 5.67 \times 10^{-8} \; L_, \; I \; \left(\frac{f}{2180}\right) \quad \text{volts}$$

L_1 = Inductance of the pick-up coil in millihenrys

I = Polarizing current in $L_,$ in amps

η = Filling factor of the pick-up coil and is defined by

$$\eta \triangleq \frac{{}_{\text{sample}} H_r^2 \sin^2 \theta \; dv}{H_r^2 \; dv} \qquad 1$$

H_r = The field at any point caused by the flow of unit polarizing current in the coil

θ = Angle between $\bar{\bar{H}}_r$ and the direction of earth's magnetic field.

This filling factor is introduced to account for the fact that all lines of flux do not cut all the turns of the coil. Therefore, some of the flux created by the precessing magnetic moments does not contribute to the voltage induced in the pick-up coil. (For the configuration of coils used in magnetometers, the value of η is about 0.3 or 0.4).

By applying certain "practical" constraints equation (1) may be plotted as $\frac{S}{N}$ versus coil size. Then it will be visibly evident just how small the coil may be made before $\frac{S}{N}$ drops below a reasonable level. These "practical" constraints were set as follows: The sample used in the coil should be spherical in shape so that at any given position in a gradient field, the value of H across the sample will not vary with orientation of the sample. Therefore the coil length and inside diameter will be equal if the sample is to just fit into the core of the coil. For a given sample diameter, the coil length and inside diameter are determined

[1]R. C. Rempel, Magnetometer Signal-to-Noise Ratio, Varian Associates Technical Memorandum TMO-23

but the outside diameter is not limited. The outside diameter should be large enough that sufficient turns can be put on the coil to give a large value of L_1, because the induced voltage from the processing protons increase directly as L_1; however, if the outside diameter is very large, the large number of turns will present a significant resistance to the polarizing current so that a large amount of power will have to be dissipated to polarize the sample, and in addition, the filling factor will be decreased. The limit of power to be put into the coil was chosen to be one hundred watts, (found from experience to be a practical upper limit.)

In order to obtain a more valid and meaningful comparison from coil to coil, the same amount of power should be put into each coil and the same amount of polarizing current used in each coil.

Since $R = \frac{\text{Power}}{I^2}$ it is evident that this can be achieved only by maintaining the same resistance from coil to coil, or what is equivalent, the same size wire and a constant volume of copper in the winding from coil to coil. With the inside diameter constrained to be the same dimension as the length of the coil, and the volume of copper constrained to be constant, the coil can be changed only by varying the ratio $\frac{\ell}{d}$ (that is, coil length divided by outside diameter.) So, when $\frac{S}{N}$ is plotted versus coil size, what will be meant by coil size is the ratio $\frac{\ell}{d}$. Plotting of equation (3) could be carried through for any size wire (provided wire size remains unchanged throughout the calculations) and the same shape of curve would be obtained.

For a constant copper volume and constant power input to the coil, the variation of $\frac{S}{N}$ with change in $\frac{\ell}{d}$ is shown in Figure 9.

The points calculated for Figure 9 do not fall exactly on a smooth

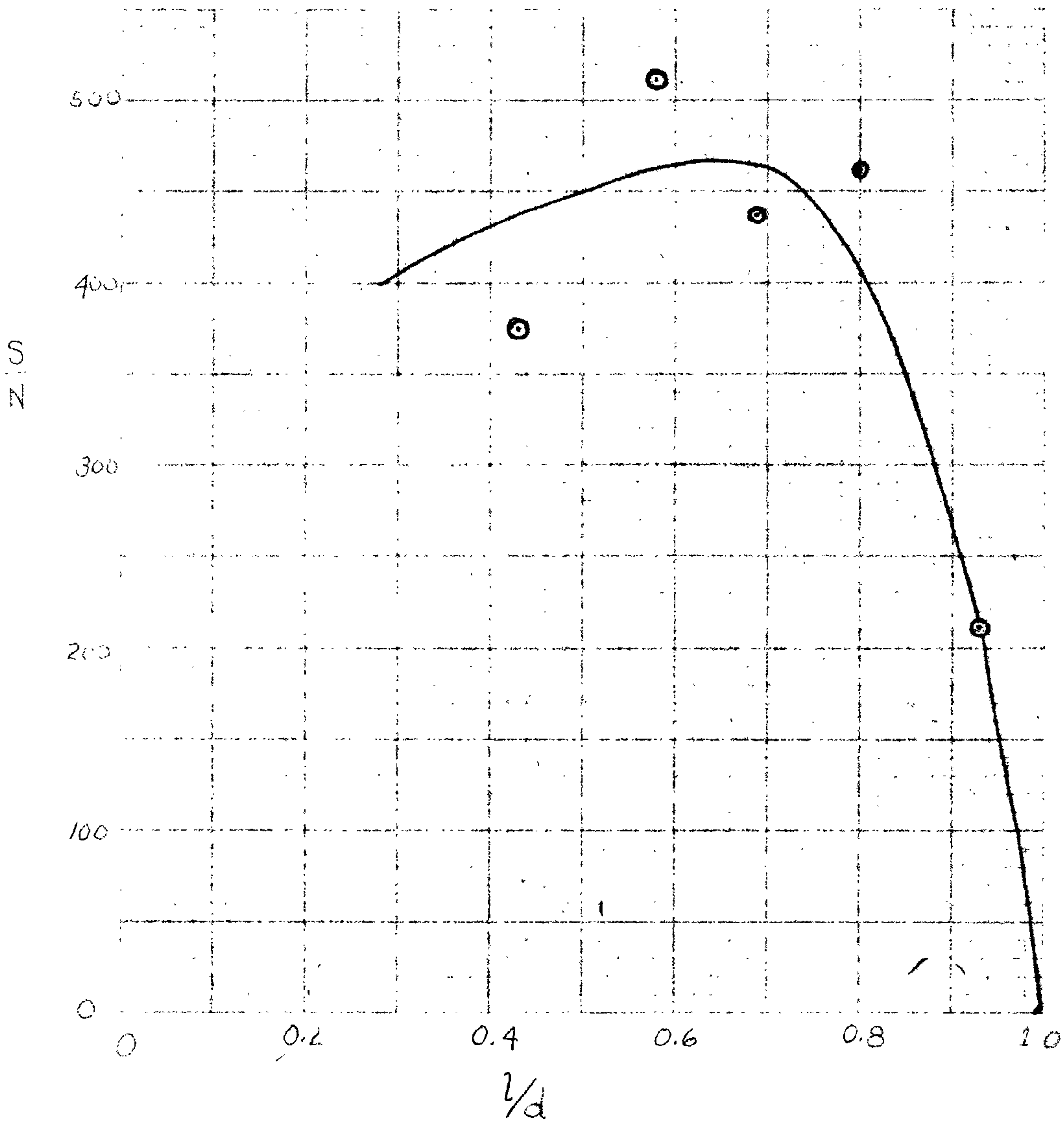


Figure 9 $\frac{S}{N}$ Ratio as a Function of $\frac{\text{coil length}}{\text{outside diameter}}$ (copper volume constant)

curve because the copper volume varied slightly from coil to coil. The reason for this is that for the calculations, a wire size was assumed and then, in order to accommodate the exact number of turns necessary for a given $\frac{\ell}{d}$ ratio, the copper volume was allowed to deviate slightly from a constant value.

Since the largest available $\frac{S}{N}$ is desired, this plot shows that the coil should be wound such that the ratio of its length to its outside diameter is approximately 0.7.

It should be noted at this point that the only noise which has been treated is the Johnson noise in the input circuit and all the noise arising from presence of the amplifier. A major source of noise which had to be neglected in the treatment so far, because of its widely varying character, was external pick-up in the sensing coil. The noises picked up by the coil result from fields created by rotating machinery, power lines, and other electronic apparatus operating in the same vicinity. These external noises greatly exceed the amplifier noise and input-circuit Johnson noise. They are counteracted by a Faraday shield and noise cancelling coil. (The use of a shield and noise cancelling coil will be elaborated on in the next sub-section.) This study of $\frac{S}{N}$ variation with coil size, however, was necessary to insure that once the external pick-up noises were counteracted, the coil would be of the proper shape and size to provide a useable $\frac{S}{N}$.

Experience has shown that a decay time, T, cannot be read if it is less than .015 seconds. The reason for this is that a transient ringing occurs when the coil is switched from "polarize" to "read," and the precession signal cannot be seen until the transient is damped out.

A practical switching and damping circuit requires long enough to damp out the transient that signals of shorter decay time than .015 seconds cannot be seen to be measured. As shown in Section 2,

$$\Delta H = \frac{2}{\gamma_p} \left(\frac{T_2}{T} - 1 \right) \frac{1}{T_2}$$

so if a plain water sample is used for which $T = 2.8$ seconds and $\gamma_p =$ 2.7×10^4 (gauss-sec)$^{-1}$, the value $T = .015$ seconds corresponds to $\Delta H \cong 500$ gamma. Preliminary measurements of field magnitude made with a standard portable magnetometer were plotted and indicated a maximum gradient of approximately one hundred gamma/inch at 40" from the mine case. The plot was extrapolated to a distance of 24" from the mine and a value of approximately 350 to 400 gamma per inch was obtained. So if the sensing head being designed must measure 400 gamma/inch the sample should not exceed $1\frac{1}{4}$" in diameter.

For a given copper volume in a coil which is constrained to carry a spherical sample in the core, the filling factor goes down as the diameter of the sample is reduced. If the filling factor is reduced too far by making the sample small in size, the coil may be ineffective in picking up a precession signal which would ordinarily be strong enough to be detected. The filling factor has previously been defined as:

$$\eta \triangleq \frac{{}_{\text{sample}}H_r^2 \sin^2\theta \, dv}{H_r^2 \, dv}$$

where H_r is the H field inside the coil from unit current flowing in the coil, and θ is the angle between the earth's field and the polarizing field. To carry through the integration would be a cumbersome task. Packard has derived the following approximate formula for the filling

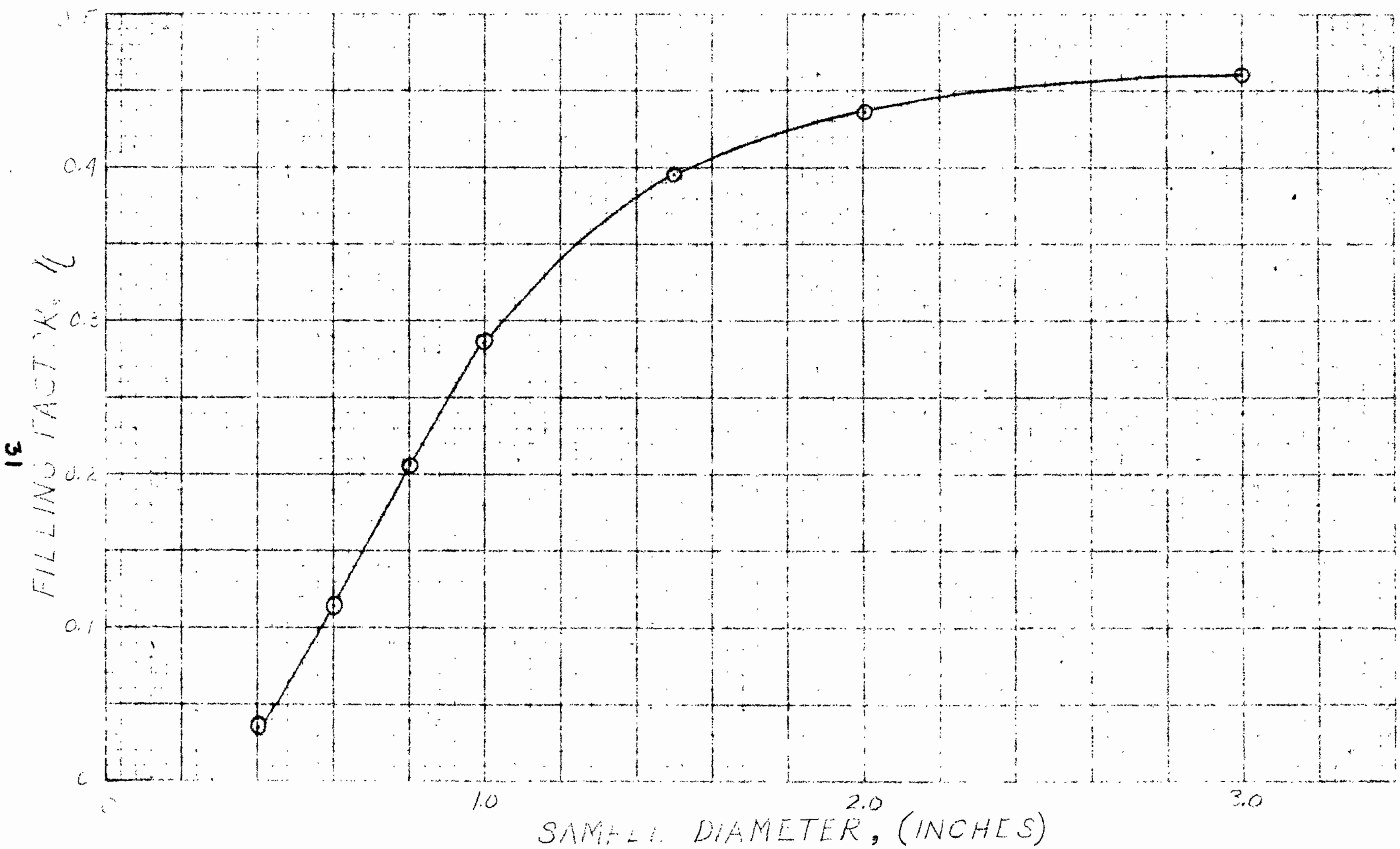


Figure 10 Coil Filling Factor as a Function of Spherical Sample Diameter (copper volume constant)

factor:[1]

$$\eta = \frac{4}{3} \; \frac{r^3_{sample}}{r^2_{coil}} \; \frac{1}{\sqrt{d^2_{coil} + {}^2_{coil}}} \qquad (2)$$

If sample diameter is equal to coil length, this reduces to

$$\eta = \frac{2}{3} \; \frac{{}^3_{coil}}{d^2_{coil}} \; \frac{1}{\sqrt{d^2_{coil} + {}^2_{coil}}} \qquad (3)$$

where d = mean coil diameter

In order to determine how small the sample size could be reduced before the filling factor limitation made further reduction impractical, equation (3) was plotted, holding copper volume constant. The value of the copper volume chosen was that for a coil of length 1" and outside diameter 1.44" (i.e., $\frac{\ell}{d} = 0.7$). The resulting plot, (Figure 10) shows that a sample diameter of 1" might be chosen as a lower limit.

Combining the consideration of $\frac{S}{N}$ ratio, filling factor, η, and decay time, T, the size of the coil capable of operating in a gradient field of 500 gamma/inch or less is obtained. From consideration of η, it should be 1" or greater in inside diameter. From considerations T, it should be 1¼" or less in inside diameter. From considerations of $\frac{S}{N}$ it should be proportioned in the ratio $\frac{\ell}{d} = 0.7$. Since it will contain a spherical sample, the length should equal the inside diameter. So a coil as shown in Figure 11 was chosen to be constructed for the experiment.

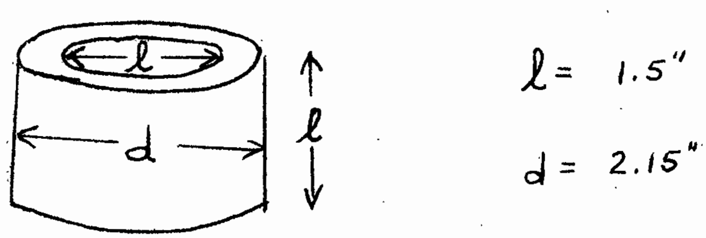


Figure 11. The Sensing Coil

[1]M. Packard "Method of Nuclear Induction": Thesis, Stanford

Although the inside diameter of the coil was $1\frac{1}{2}''$, the sample was contained in a glass approximately 1/8" in thickness, so the diameter of the sample was $1\frac{1}{4}''$.

It can be seen from the foregoing calculations that the coil size chosen is not the absolute minimum that would work. Smaller coils should be capable of producing a signal due to nuclear induction, but this size coil appears from the calculations to offer an optimum for the conditions under which it must operate.

The size of wire with which to wind the coil was chosen to give practical values of current and inductance. The Varian Station Magnetometer operates on 5 amps polarizing current. If this smaller coil is wound using number 20 copper wire, the d. c. resistance would be about 2.5 ohms so that with a 12 volt storage battery used as the polarizing source, a current of slightly under 5 amps would flow in the coil. The storage battery is capable of providing this amount of current for a reasonable period of time. Although the single coil with its sample constitute the whole sensing head, an additional coil is added. This is a noise cancelling coil to aid in reducing the noise pick-up when working in the presence of a number of operating electronic devices and electrical machinery. Both coils are further surrounded by a Faraday shield. The noise cancelling coil is identical with the sensing coil. It is placed beside the sensing coil with the axis of the two parallel and connected in series such that an external field will induce currents in each coil which flow in opposite directions and therefore cancel. Besides the added size of the sensing head when a noise cancelling coil is employed, twice as much power must be supplied, because both coils must be polarized.

This is no problem, though, if the power is available. Instead of one 12 volt battery, two were used as the polarizing source.

The electrical characteristics of the finished double coil were:

$L_1 = L_2 = 6.91$ Mh

$L_1 + L_2 + 2M = 14.6$ Mh

Rdc = 4.7 ohms

Q =42.5

(It should be noted that although the two coils are connected in series opposing so far as external fields are concerned, they are in series aiding with respect to the field produced by the opposite coil, and so the total inductance is the sum of the separate inductances plus two times the mutual inductance.)

The water sample was placed into one of the coils in a spherical glass bulb. A sample could be placed in both coils and the output voltage would be doubled. However the characteristic exponential envelope of the output would be observed only if the two coils were in the same value of magnetic field. If they were not, the frequency of the voltage induced in each would be different, and the resultant output would have an envelope containing beats at the difference frequency. Since the time constant of the decay of such a signal would be difficult to measure, a sample was used in only one coil.

c. The Experimental System

The system that was set up for taking the data to produce a plot of the gradients around the mine is shown in Figure 12.

An explanation of the various elements of the system is given below.

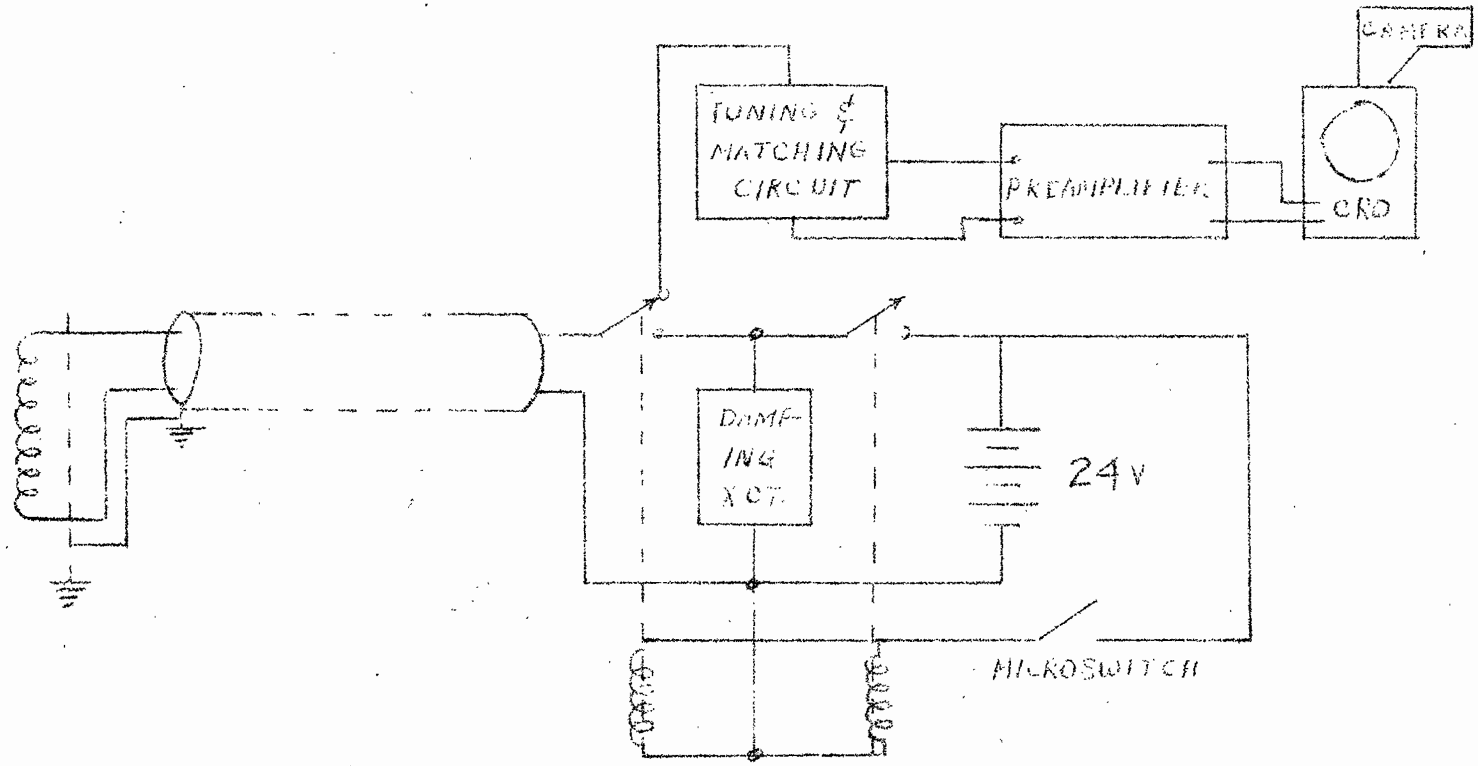


Figure 12 Simplified Diagram of the Experimental Equipment

(1) Preamplifier

The preamplifier is a three stage transistorized audio amplifier. The only reason transistors were used was that eventually, in the final package, the preamplifier would have to be located right with the sensing head. This would be necessary because of the attenuation in the length of cable between the sensing element and the ship. The 3 db bandwidth of the preamplifier is 4137 c.p.s. (from 363 c.p.s. to 4500 c.p.s.). It was made fairly broad band to handle the eventuality of fields much different from 50,000 gamma as the result of large gradients. However, this bandwidth was later found unnecessary and so a narrow bandpass filter was added after the third stage to reduce noise, and the input was tuned to enhance the input signal.

The measured characteristics of the preamplifier are as follows:

Gain = 85 db (without input tuning)

Gain =112 db (with tuning)

Bandwidth = 4137 c.p.s. (363 to 4500)

Noise Figure = 8.43 db

A circuit diagram of the preamplifier is shown in Figure 13.

(2) Tuning and Matching Network

Since the value of magnetic field around the mine did not vary enough to require a wide bandwidth system, the input signal to the preamplifier could be increased by tuning the pick-up coil to approximately 2180 c.p.s. (the frequency

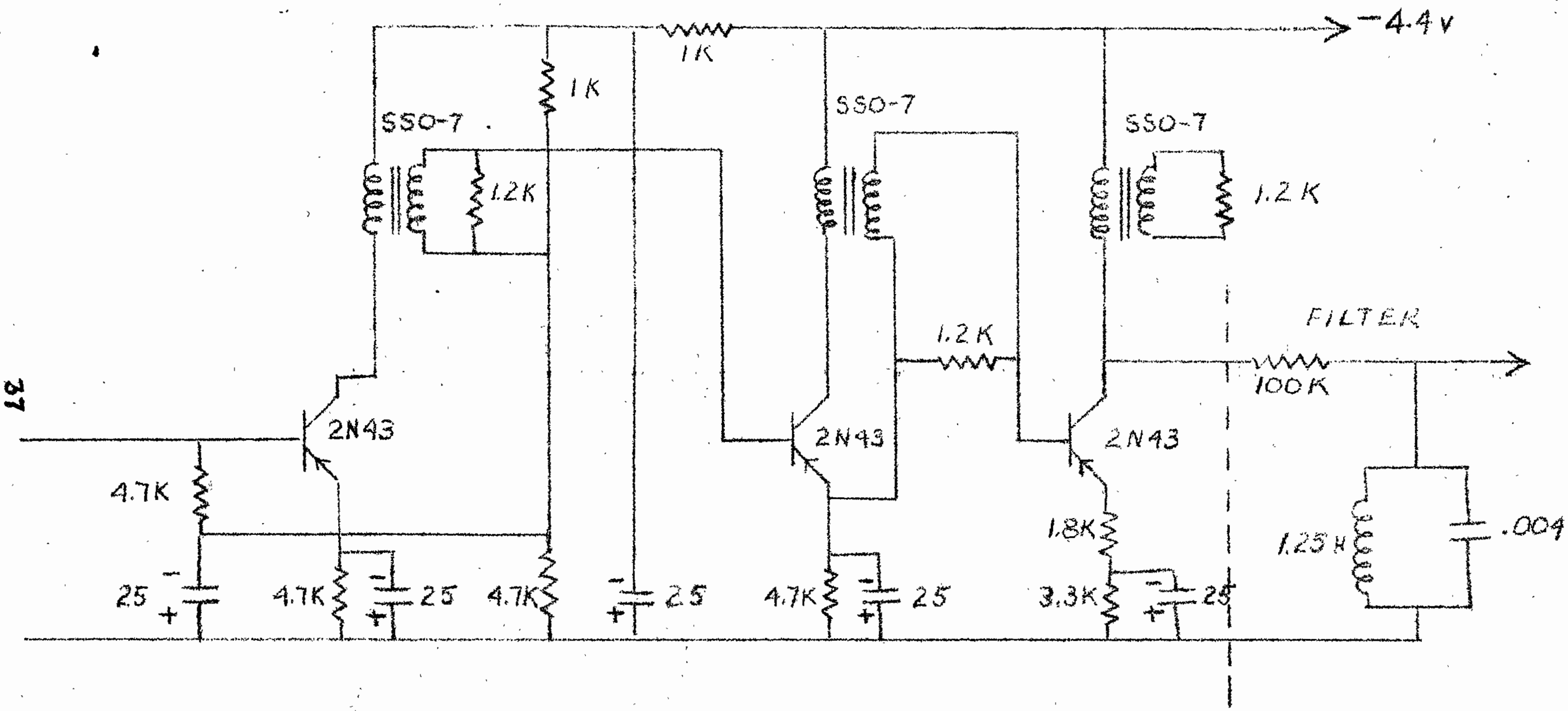
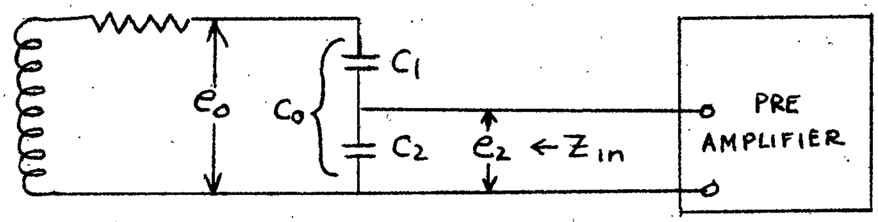


Figure 13 Preamplifier and Output Filter

corresponding to a field of $\frac{1}{2}$ gauss.) This served to increase the $\frac{S}{N}$ ratio of the system. When a capacitor was placed in parallel with the sensing coil to tune it, the impedance looking into the combination did not match the preamplifier input impedance of approximately 800 ohms. The impedance match was accomplished by a capacitor divider arrangement:



$C_o = .36\ \mu f$

$$Z_{in} = \left(\frac{e_2}{e_o}\right)^2 Z_o \qquad \text{where } Z_o = Q L \omega$$

$$= \frac{L}{RC_o}$$

$$Z_{in} = 800 = \left(\frac{C_o}{C_2}\right)^2 Z_o \qquad = 8630\ \Omega$$

$$C_2^2 = \frac{.1295}{800}(8630)$$

$$= 1.4$$

$$C_2 = 1.18\ \mu f$$

$$C_1 = 0.52\ \mu f$$

The series combination of C_1 and C_2 is the capacitance necessary to resonate the coil at 2180 c.p.s. C_2 is of such a value that looking back into the L-C combination across C_2 to ground, the amplifier sees 800 ohms impedance.

(3) Relay and Damping Circuit

Since the same coil is used to pick up the nuclear precession signal and to polarize the sample, a switching arrangement must be provided to switch the coil between the polarizing source and the preamplifier. This problem is slightly complicated by the fact that the coil cannot be connected instantaneously to the preamplifier once the polarizing voltage is disconnected. The polarizing current that continues to flow in the coil is of sufficient magnitude that it would damage the transistors of the preamplifier, and so a delay must be provided between the time the coil is disconnected from the polarizing source and the time it is connected to the preamplifier. During this delay, the current remaining in the coil must be damped out. It is desirable to cut the current off as quickly as possible or protons which were lined up with the polarizing field will precess about the decaying field at various frequencies corresponding to the values assumed by this decaying field rather than precessing about the earth's magnetic field at a single frequency and in a coherent manner.

To accomplish the desired effect, two switches are used. The first disconnects the polarizing voltage from the coil permitting the current to flow into a damping circuit. After the current has been damped to a sufficiently low value, a second switch disconnects the coil from the damping

circuit and connects it to the preamplifier.

A circuit diagram of the switching relay and damping system is shown in Figure 14.

d. Collecting the Data for Gradient Pattern With Mine Oriented North-South

To measure the gradients in the earth's field in a plane 24" above the upper surface of the mine, the setup shown in Figure 15 was used.

The mine was placed in an open space out-of-doors away from any other metal objects that might affect the gradient in the earth's field significantly. Then the coil was moved to various positions on the stand over the mine, and at each position a picture was taken of the waveform on the CRO resulting from the proton precession signal. Two examples of such pictures are shown in Figure 1 on page 3. The time of decay of the signal was measured on the photograph, and this time converted to the gradient in gamma/inch. A plot of the gradients measured in a plane 24" above the mine case is shown in Figure 16.

From the plot of Figure 16, a simulated display was made as the mine would be viewed on an oscilloscope. This display was constructed as though a number of measurements were taken simultaneously by a matrix of sensing heads towed across the mine 24" above it, and a spot appeared on the scope for each sensing head. If the intensity of each spot were proportional to the gradient at the point it represents, the presentation seen in Figure 17 would be obtained.

e. Gradient Pattern With Mine Oriented East-West.

The plot of the gradient field above the mine with it oriented in a North-South direction turned out to be approximately what was expected and contained several characteristics that could certainly identify it.

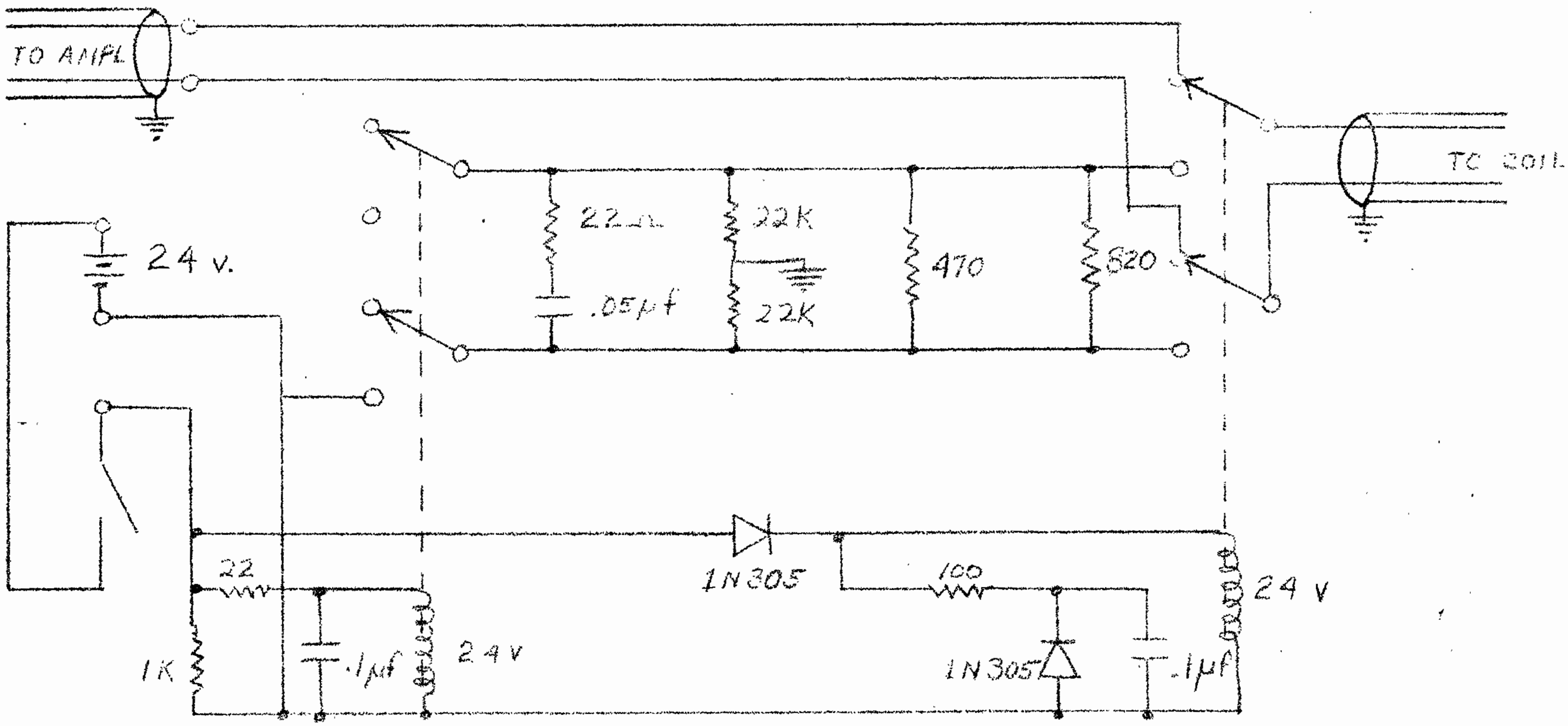


Figure 14 Switching Relays and Damping Circuit

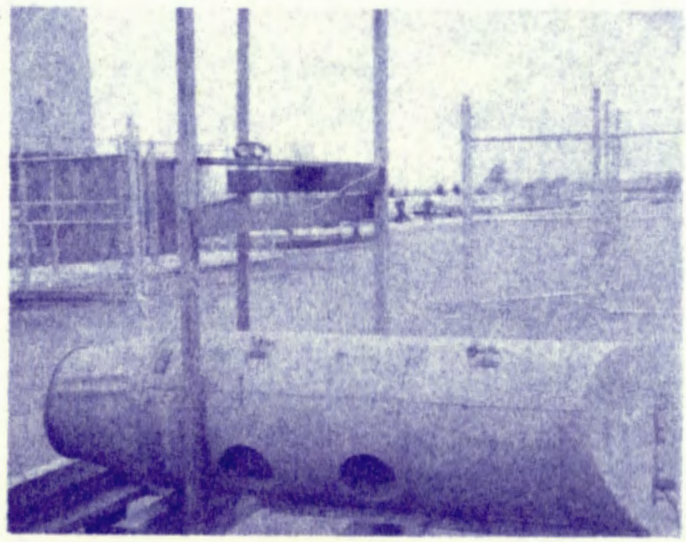


Figure 15 Mk. 25-Mod 1 Mine Case with Gradient Measuring Coil in Position for Measurement.

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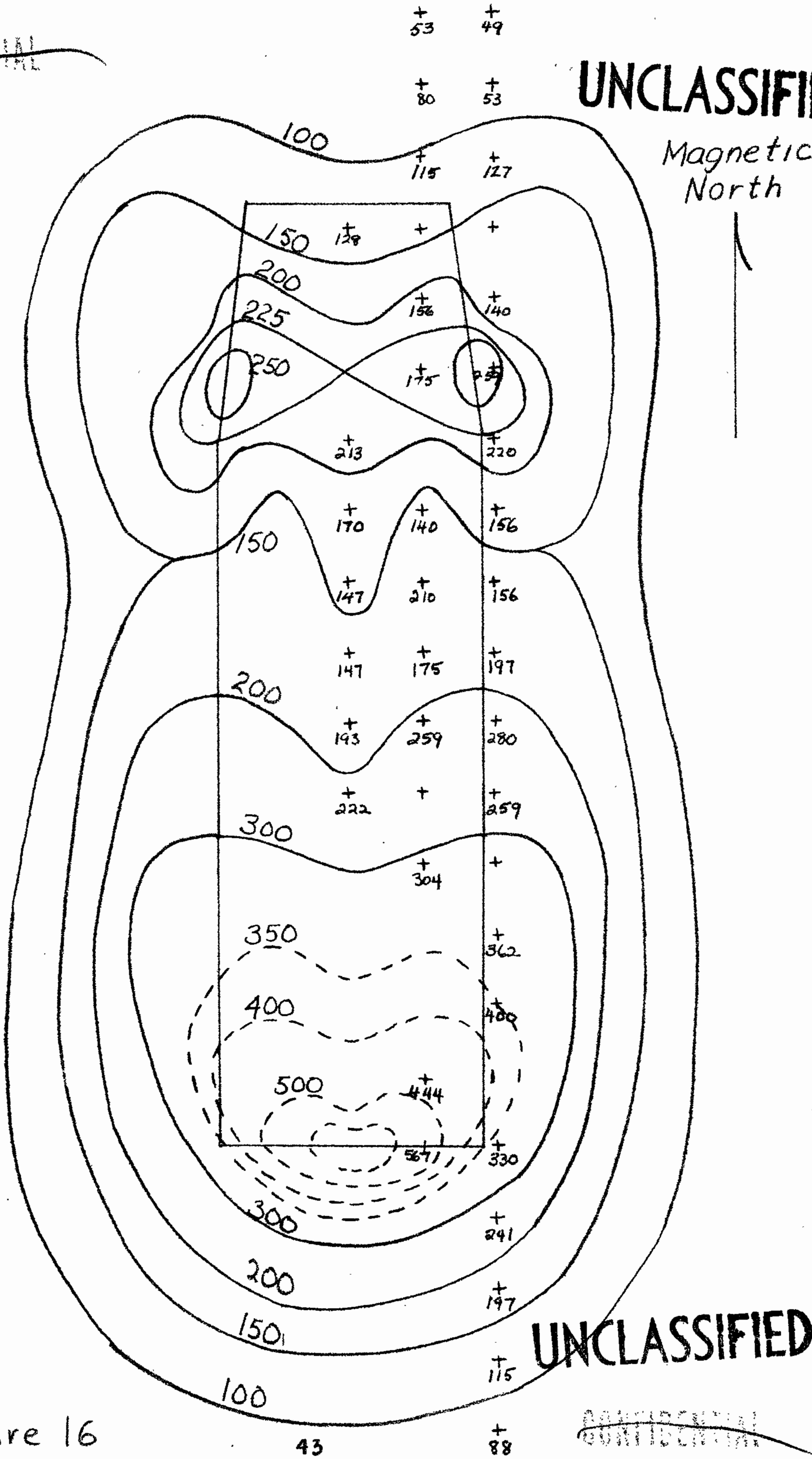

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Figure 16

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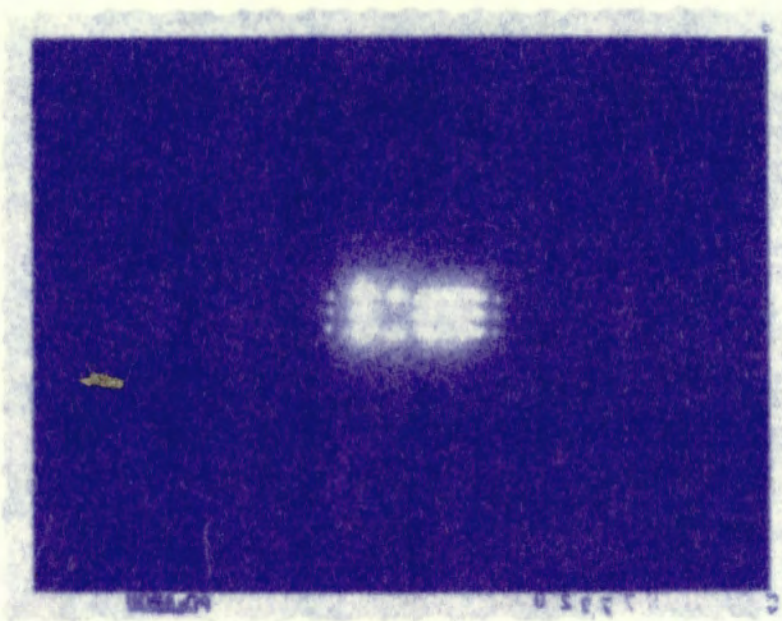

Figure 17

The next step in the experiment was to orient the mine east-west and repeat the measurements to determine if an entirely different pattern is obtained. If the pattern obtained with the east-west orientation is significantly different from that obtained with a north-south orientation, this method of identifying mines will not prove practicable.

Before taking measurements on the mine in an east-west orientation, it would be profitable to try to determine what sort of a pattern to expect. In the first place, the induced magnetism should be expected to give an effect the same as that shown in Figure 6 (i.e., there should be higher gradients at the north and south sides of the mine case and lower gradients between). However, since the distance across the mine in the direction of the earth's magnetic field is less in the east-west orientation than in the north-south orientation the effect might be expected to be less. (This actually turns out to be the case; the value of gradients measured for the east-west orientation are about one-half the value of the corresponding measurements in the north-south orientation.)

Secondly, if there is any permanent magnetism in the mine case it should be expected to appear as a certain characteristic in the plot of the gradient field, and this characteristic should be present both in the north-south orientation plot and the east-west orientation plot. For this reason, we should expect the east-west plot to bear at least some resemblance to the north-south plot.

Our conclusions, then, are that the east-west plot should be similar in shape to the north-south plot but the gradient values less than in the north-south plot. Figure 18 is a plot of the gradient field for the mine case oriented in an east-west direction. It is seen to take on a similar,

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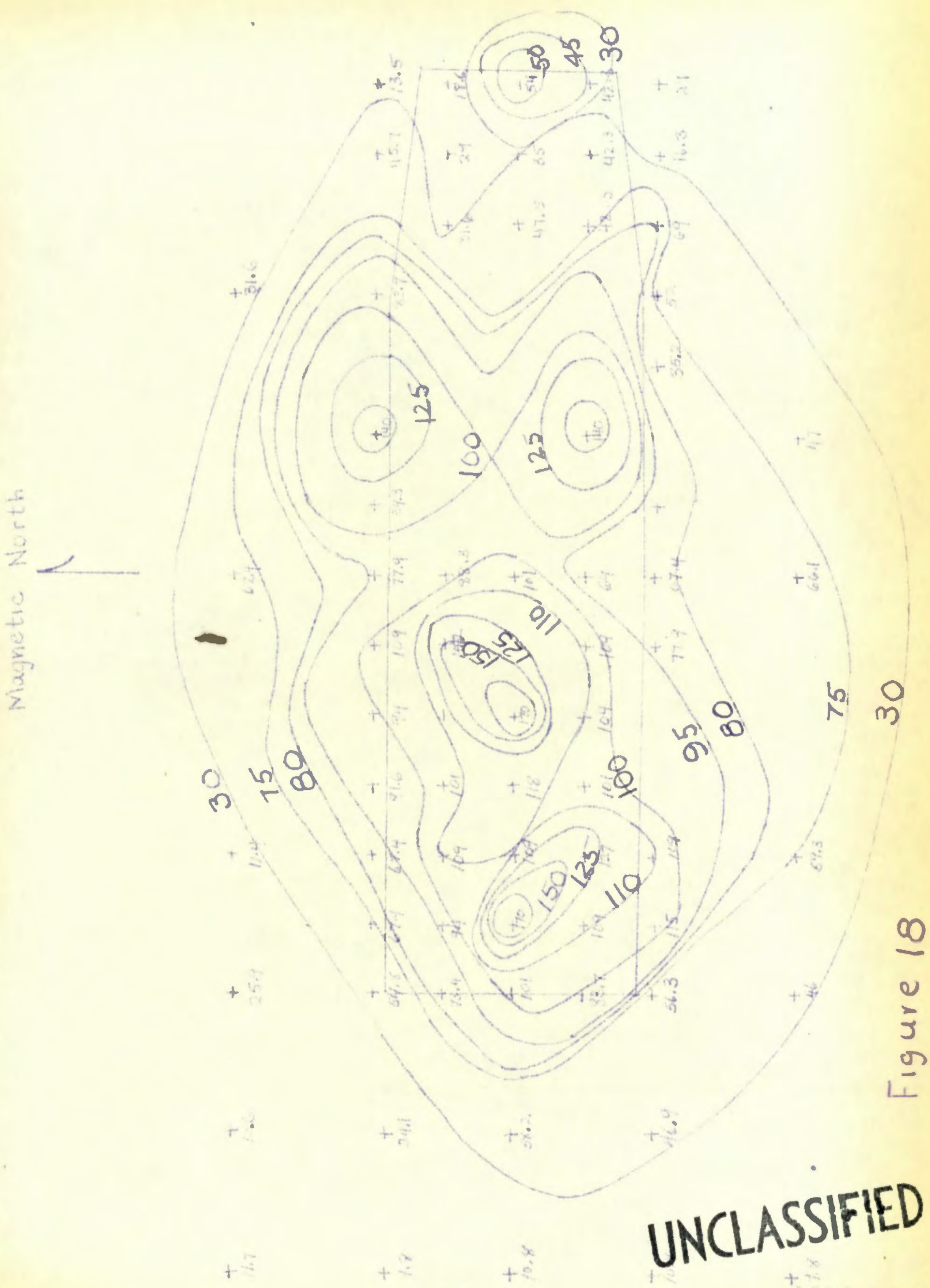

Figure 18

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(but distorted) pattern, this similarity is presumably due to the permanent magnetism of the mine case, and it is seen that the gradient values are approximately half the value of the corresponding gradients in the north-south orientation pattern.

When the gradients were measured in the east-west orientation, an experiment was conducted to find if a still smaller coil could be used. A new coil was wound which was half the size of the coil used for measurements in the north-south orientation; that is, the inside diameter was 3/4" and the total inductance of the sensing coil and noise cancelling coil in series was 7.1 Mh. The following modification to the input circuit was made:

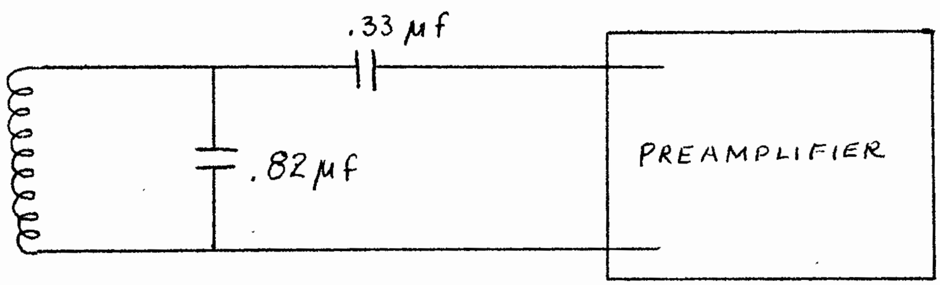
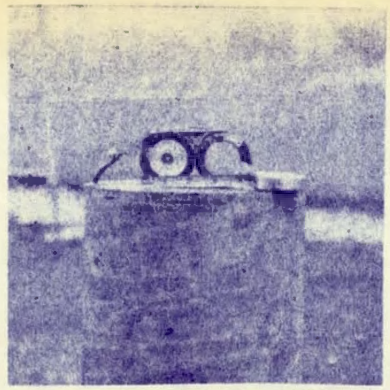
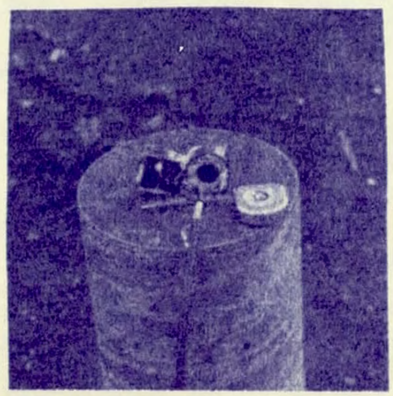


Figure 19. Modified Input Circuit

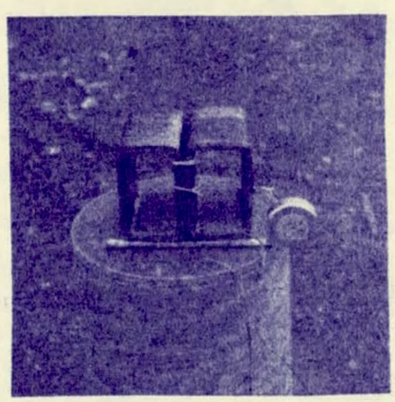
The polarizing source was increased to 30 volts to provide a polarizing current of 8 amps, and the sample was polarized a second or two longer for each reading to obtain a useable magnitude of output signal. The volume of the sample had been reduced by a factor of four in this smaller coil so these steps were necessary to align enough nuclear moments in the direction of the polarizing field to get a useable signal. Figure 20 b shows a photograph of the 3/4" coil, along with the one used for north-



(a) $1\frac{1}{2}''$ (inside diameter) Coil



(b) $\frac{3}{4}''$ (inside diameter) Coil



(c) Station Magnetometer Coil

Figure 20 Photographs of Coils Used for Gradient Measurements and Station Magnetometer Coil, Showing Relative Sizes.

south measurements, Figure 20 a, and a station magnetometer coil, Figure 20 c for size comparison

The smaller 3/4" coil did not exhibit any advantages over the $1\frac{1}{2}$" coil. It should have been capable of readings closer to the mine, but for unexplainable reasons would not produce a signal any closer than the $1\frac{1}{2}$" coil. The 3/4" coil had the major disadvantage of heating. The transverse relaxation time, T_2, of any particular substance increases with temperature, and so care had to be exercised that readings were taken at about the same temperature and the T_2 of the sample had to be measured at the operating temperature. It was concluded that the $1\frac{1}{2}$" coil is the more nearly optimum size of the two coils.

4. Conclusions.

The relatively small amount of data obtained in this experiment limits the conclusions which can be reached. Only one mine was tested and it gave a pattern which, if seen again, might be very easily identified as a Mk .25 type mine case. It should not be concluded from this that the field gradient pattern scheme for identifying mines will work in all cases. For one thing, it is not known what difference in pattern would be obtained if there were no permanent magnetism in the mine case or if the permanent magnetism in another Mk .25 type mine case could differ enough to give an entirely different pattern.

However, the results of this experiment can lead to some positive conclusions regarding the classification of mines by a system utilising the field gradient pattern scheme. The fact that the one mine tested did have an identifying signature is promising enough to make further investigation seem profitable. Then, what may prove more important, the experiment shows that even if a given class of mines does not always yield the same pattern, a fairly good estimate of two of the dimensions of the mine can be obtained. In the two plots obtained, if the contour is arbitrarily chosen whose value is 1/3 the maximum gradient measured (say 200 gamma/inch for the north-south plot and 50 gammas per inch for the east-west plot) it shows the mine to be 80" x 40" and 75" x 32" respectively as compared with the actual dimensions 80" x 22". Even though this gives distorted dimensions, it compares favorably with a high resolution sonar.

Since the dimensions of the mine cannot be obtained this accurately by presently available sonar, it can be concluded that the gradient

pattern method provides additional information over that available from sonar. These dimensions are, of course, information that would help identify a contact as mine or non-mine.

The conclusions that may be reached as a result of the foregoing experimentation are, then, as follows:

1. The method of mine classification by the magnetic field gradient pattern scheme appears promising because it presents more information than is now available from sonar.

2. There is not enough information available to tell whether a given <u>class</u> of mine has a characteristic signature, but it appears possible to at least measure the size of the mine in two dimensions to a fair degree of accuracy.

BIBLIOGRAPHY

1. Varian Associates Technical Proposal 577 dated March 11, 1957, A Magnetometer for the Classification of Sea Mines. (Confidential)

2. Charles H. Bowen, An Earth's Field Magnetometer that Utilizes the Free Precession of Protons, (Thesis) United States Naval Postgraduate School, 1954.

3. Henry Semat, Introduction to Atomic and Nuclear Physics, Rinehart and Co., Inc., New York, 1939.

4. F. Bloch, W. W. Hansen, M. Packard, The Nuclear Induction Experiment Phys. Rev., 70, p. 474 Oct. 1946.

APPENDIX I

Derivation of an expression for the voltage induced in a coil by nuclear induction from spins in a spherical sample which is in a magnetic field of constant gradient.

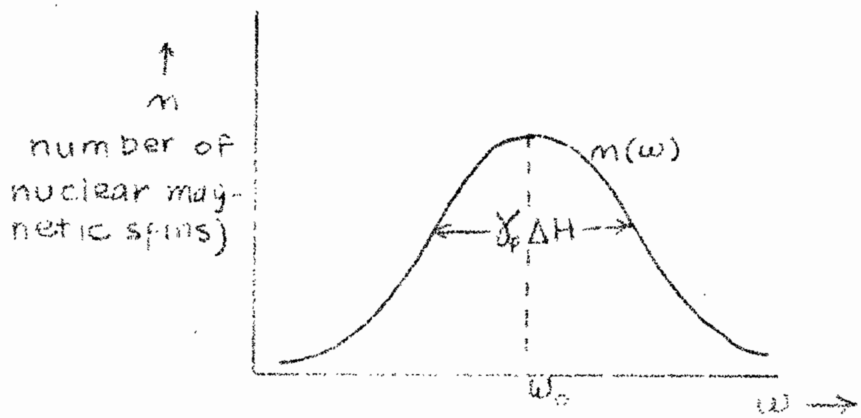


Figure I-1 Distribution of spins subjected to magnetic field.

Each nuclear moment, as it precesses about the direction of the earth's magnetic field induces a voltage of the form $\varepsilon^{j\omega t}$ in a properly oriented pick-up coil. If the number of spins subjected to a value of magnetic field (or the value of ω corresponding to that field H, since $\omega = \gamma_p H$) is as shown in Figure I-1, the voltage induced by the moments between ω and $\omega + d\omega$ is proportional to $m(\omega)\,\varepsilon^{j\omega t}\,d\omega$, assuming all the spins to be precessing in phase. The total induced voltage from all the spins in a sample is

$$f(t) = \int m(\omega)\,\varepsilon^{j\omega t}\,d\omega$$

Assume that the gradient of the field across the sample is such that the number of spins subjected to a given field assumes a Lorentzian distribution:

$$m(\omega) = \frac{M}{1 + (\omega - \omega_0)^2 \left(\frac{2}{\gamma_p \Delta H}\right)^2}$$

(For the case of a constant field gradient across a spherical sample, this is a good approximation. This fact, coupled with the fact that the use of the Lorentzian distribution leads to an exponential expression and so, in this case, simplifies the mathematics, seems sufficient justification for its use.)

The total induced voltage would then be

$$f(t) = \int_{-\infty}^{\infty} \frac{M \mathcal{E}^{j\omega t}}{1 + (\omega_0 - \omega)^2 \left(\frac{2}{\gamma_p \Delta H}\right)^2} \, d\omega \qquad (1)$$

Define

$$(\omega_0 - \omega) = \Delta\omega$$

Then $\omega = \omega_0 - \Delta\omega \qquad d\omega = d(\Delta\omega)$

Substituting into (1)

$$f(t) = \int_{-\infty}^{\infty} \frac{M \mathcal{E}^{j(\omega_0 - \Delta\omega)t}}{1 + (\Delta\omega)^2 \left(\frac{2}{\gamma_p \Delta H}\right)^2} \, d(\Delta\omega) \qquad (2)$$

$$= \frac{M \mathcal{E}^{j\omega_0 t}}{\left(\frac{2}{\gamma_p \Delta H}\right)} \int_{-\infty}^{\infty} \frac{\mathcal{E}^{-j(\Delta\omega)t}}{1 + (\Delta\omega)^2 \left(\frac{2}{\gamma_p \Delta H}\right)^2} \, d(\Delta\omega)$$

$$= \frac{M \mathcal{E}^{j\omega_0 t}}{\left(\frac{2}{\gamma_p \Delta H}\right)} \int_{-\infty}^{\infty} \frac{\exp -j\left(\Delta\omega \frac{\frac{2}{\gamma_p \Delta H}}{\frac{2}{\gamma_p \Delta H}}\right)t}{1 + (\Delta\omega)^2 \left(\frac{2}{\gamma_p \Delta H}\right)^2} \, d\left(\Delta\omega \frac{2}{\gamma_p \Delta H}\right)$$

Equation (3) is of the form

$$K \int_{-\infty}^{\infty} \frac{\mathcal{E}^{-jx}}{1 + x^2} \, dx = K \int_{-\infty}^{\infty} \frac{\cos x}{1 + x^2} \, dx - j K \int_{-\infty}^{\infty} \frac{\sin x}{1 + x^2} \, dx$$

In the imaginary half, the numerator is an odd function and the denominator is an even function, so the result of the integration of the imaginary part from $-\infty$ to $+\infty$ will be zero.

From integral tables

$$\int_0^\infty \frac{\cos mx}{1+x^2}\,dx = \frac{\pi}{2}\,\varepsilon^{-|m|}$$

$$\therefore \quad \frac{M\varepsilon^{j\omega_0 t}}{\left(\frac{2}{\gamma_p \Delta H}\right)} \int_{-\infty}^{\infty} \frac{\exp -j\left(\Delta\omega \frac{\frac{2}{\gamma_p\Delta H}}{\frac{2}{\gamma_p\Delta H}}\right)t}{1+(\Delta\omega)^2\left(\frac{2}{\gamma_p\Delta H}\right)^2}\, d\left(\Delta\omega\frac{2}{\gamma_p\Delta H}\right)$$

$$= \frac{M\varepsilon^{j\omega_0 t}}{\left(\frac{2}{\gamma_p\Delta H}\right)}\,\pi\,\varepsilon^{-\frac{t}{\frac{2}{\gamma_p\Delta H}}}$$

$$\triangleq \frac{M\varepsilon^{j\omega_0 t}}{T_2^*}\,\pi\,\varepsilon^{-\frac{t}{T_2^*}}$$

So

$$f(t) = \frac{M\varepsilon^{j\omega_0 t}}{T_2^*}\,\pi\,\varepsilon^{-\frac{t}{T_2^*}} \tag{4}$$

This derivation does not take into account the natural decay time constant, T_2, due to the spin-spin and spin-lattice couplings in the sample. Because of the presence of these couplings, the induced voltage from a sample in a zero gradient field includes an attenuation term $\varepsilon^{-\frac{t}{T_2}}$ where T_2 is a constant of the sample material. Therefore the expression for the voltage induced from a sample in a non-zero gradient field contains both the attenuation factor $\varepsilon^{-\frac{t}{T_2}}$ and $\varepsilon^{-\frac{t}{T_2^*}}$. The induced voltage V, is proportional to

$$V \sim \varepsilon^{-\frac{t}{T_2}}\,\varepsilon^{-\frac{t}{T_2^*}}\,\varepsilon^{j\omega_0 t}$$

or

$$V \sim \varepsilon^{-\frac{t}{T}}\,\varepsilon^{j\omega_0 t}$$

where $$\frac{1}{T} = \frac{1}{T_2} + \frac{1}{T_2^*}$$

T_2 = the natural decay time of the sample

T_2^*= the decay time due to the presence of a field gradient

Equation (2) may be rewritten in the form

$$f(t) = \frac{M\varepsilon^{j\omega_0 t}}{\left(\frac{2}{\gamma_p \Delta H}\right)^2} \int_{-\infty}^{\infty} \frac{\varepsilon^{-j(\Delta\omega)t}}{\left(\frac{\gamma_p \Delta H}{2}\right)^2 + (\Delta\omega)^2} \, d(\Delta\omega)$$

For an actual sample, the integration would not be taken over all ω but just over the limits of ω actually existing across the sample (say ω_1 to ω_2.) Therefore the expression for the voltage would take the form:

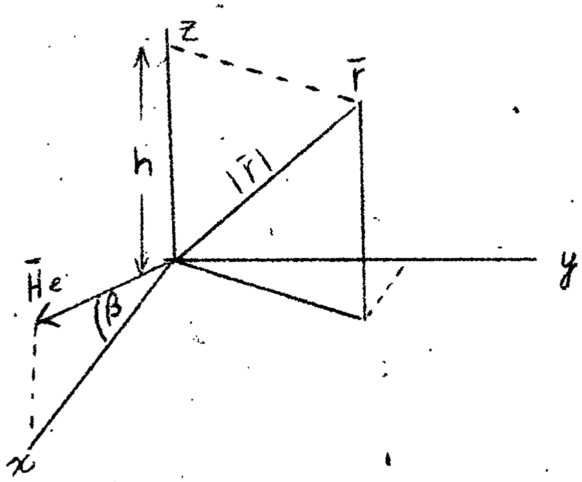
$$f(t) = \frac{M\varepsilon^{j\omega_0 t}}{\left(\frac{2}{\gamma_p \Delta H}\right)^2} \int_{\omega_1}^{\omega_2} \frac{\varepsilon^{-j(\Delta\omega)t}}{\left(\frac{\gamma_p \Delta H}{2}\right)^2 + (\Delta\omega)^2} \, d(\Delta\omega)$$

When expressed in this form, it is seen that the voltage induced by the nuclear spins in a sample located in a gradient field is proportinal to the Fourier Transform of a function of the change of field across the sample.

APPENDIX II

Derivation of an expression for the magnetic field in a plane above an induced magnetic dipole.

A coordinate system is defined with z as the vertical direction, x the direction of the horizontal component of the earth's magnetic field, and y completing the righthand orthogonal system. The origin is taken as the location of the dipole.



The total field in the region well away from the material is

$$\bar{H} = \bar{H}e + \bar{H}d$$

where $\bar{H}d$ is the field induced by the dipole moment $\bar{m}$. This field is

$$\bar{H}d = \nabla\left[m \cdot \nabla\left(\frac{1}{r}\right)\right] = \frac{\bar{m}\, r^2 - 3\bar{r}\,(\bar{m}\cdot\bar{r})}{r^5}$$

Let $\bar{H}_{de}$ be the component of $\bar{H}_d$ in the direction of $\bar{H}e$ and $\bar{H}_{dt}$ be the component transverse to $\bar{H}e$. At distances such that $Hd \ll He$

$$H = |\bar{H}| = \left[(He + Hde)^2 + Hdt^2\right]^{1/2}$$

$$H = He\left[1 + \frac{Hde}{He} + \frac{Hdt}{2He^2} + \cdots\right]$$

$$H \approx He + Hde$$

$\bar{m}$ will be predominately in the direction of $\bar{H}_e$; when it is aligned exactly with $\bar{H}_e$,

$$H_{de} = \frac{m}{r^3}\left(1 - 3\cos^2\alpha\right)$$

where α = angle between $\bar{H}_e$ and $\bar{r}$. ($\bar{r}$ points from the induced dipole to the point where the field is being measured.)

Since $\bar{H}_e \cdot \bar{r} = H_e r \cos\alpha$

$$\cos\alpha = \frac{x\cos\beta}{r} - \frac{h\sin\beta}{r}$$

Substituting (2) into (1)

$$H_{de} = \frac{m}{r^3}\left\{1 - 3\left[\frac{x^2\cos^2\beta}{r^2} + \frac{h^2\sin^2\beta}{r^2} + \frac{xh\sin 2\beta}{r^2}\right]\right\}$$

If the dipole is passed over at a height h while travelling in the north-south direction, $y = 0$ and

$$H_{de} = \frac{m}{(x^2+h^2)^{3/2}}\left\{1 - 3\left[\frac{x^2\cos^2\beta - xh\sin 2\beta + h^2\sin^2\beta}{x^2+h^2}\right]\right\}$$

and the gradient in the direction of travel is

$$\frac{\partial}{\partial x} H = \frac{\partial}{\partial x}\left(H_e + H_{de}\right) = \frac{\partial H_e}{\partial x}$$

$$= -\frac{3m}{(x^2+h^2)^{3/2}}\left\{x\left(1+2\cos^2\beta\right) - h\sin 2\beta - 5x\left(\frac{x^2\cos^2\beta - xh\sin 2\beta + h^2\sin^2\beta}{x^2+h^2}\right)\right\}$$

For the simple case in which $\beta = 0$, if $\frac{\partial H_e}{\partial x}$ is set equal to zero, it is found that the extremes of H_e occur at $x = \pm Kh$ which agrees with the results obtained in Figure 6.